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M. M. Weiner

MTR 92B0000090
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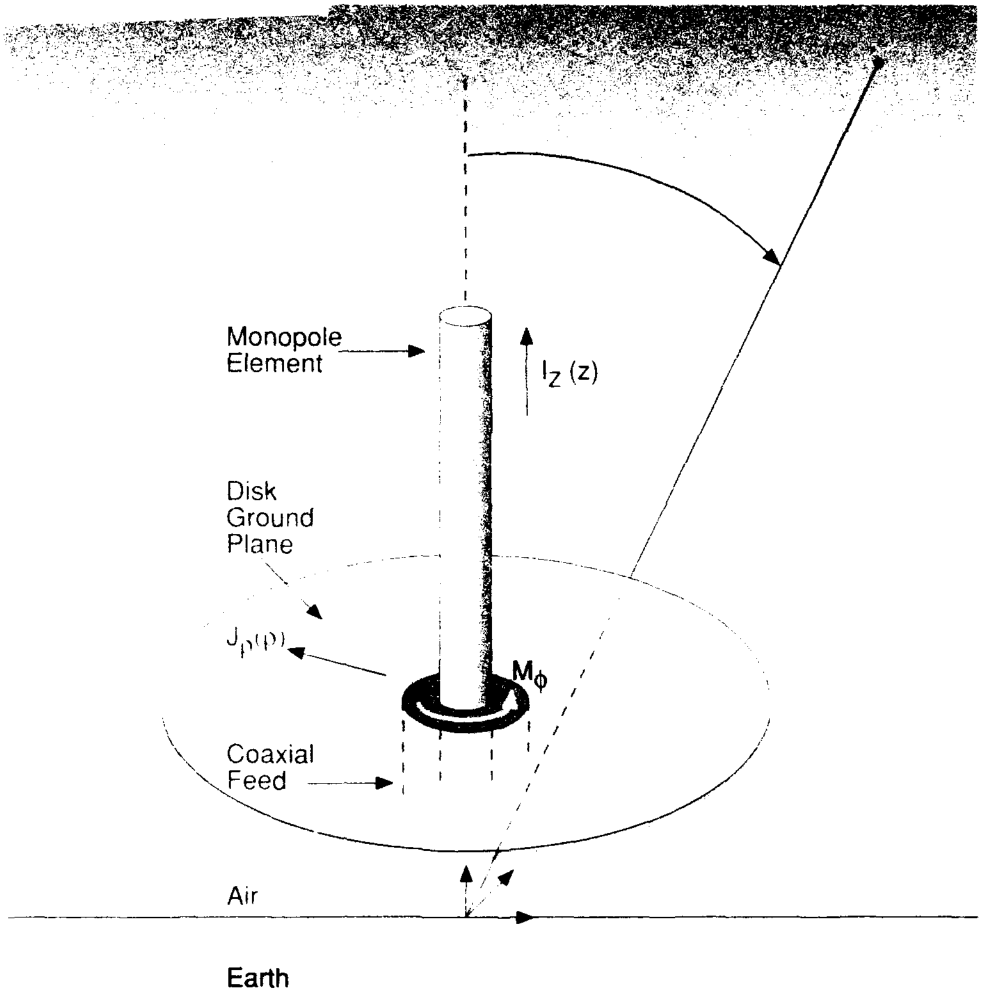

MITRE
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ABSTRACT

Richmond's moment-method analysis for the current distribution and input impedance of a monopole element on a disk ground plane above flat Earth is used to obtain the far-zone field in the free-space region. Numerical results for directivity and radiation efficiency are presented as separate entities, unlike previously reported results based on Monteath's compensation theorem or Sommerfeld's attenuation function that give only the product of the directivity and radiation efficiency.

ACKNOWLEDGMENTS

The theory is based on a report written by Dr. Jack H. Richmond (deceased) of Ohio State University when he was a member of the technical advisor group to the MITRE sponsored research project 91260 "High-Frequency Antenna Element Modeling," Melvin M. Weiner, Principal Investigator. Dr. Richmond developed the theory and computer program RICHMOND4 for the far-zone field. Christopher Sharpe and Enis Vlashi performed the computer runs and obtained the computer plots. Elinor Trottier and Sheila Lamoureux typed the manuscript.

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SECTION 1

INTRODUCTION

The modeling of monopole elements with circular ground planes in proximity to Earth has been greatly enhanced in recent years by method-of-moments programs developed by Richmond for disk ground planes [1] and by Burke, et al., for radial wire ground planes [2,3,4].

The method-of-moments models include the following advantages over models based on Monteath's compensation theorem [5,6,7,8,19] or Sommerfeld's attenuation function [9]: (1) current on the ground plane is computed rather than approximated by that for a perfect ground plane; (2) results are valid not only for moderately large ground planes, but for electrically small ground planes; (3) ground-plane edge diffraction is determined directly rather than neglected or obtained by perturbation methods; (4) analytical restrictions on evaluating Sommerfeld's integral (such as requiring that the Earth's complex relative permittivity have a modulus much greater than *unity*) are avoided; and (5) directivity and radiation efficiency are determined as separate entities, rather than being lumped together as a product to yield the antenna gain. Nevertheless, those other models are useful for validating method-of-moments numerical results and for treating large ground planes whose segmentation in method-of-moments models would exceed computer computational capacity and precision.

Richmond has presented a moment-method analysis for the current distribution and input impedance of a monopole element on a disk ground plane in free-space [10] and above flat Earth [1] with numerical evaluation by computer programs RICHMD1 and RICHMD3, respectively. Weiner, et al. [11,12] have used Richmond's results in reference 10 to develop a computer program, RICHMD2, for the far-zone field, directivity, and radiation efficiency for the case when the ground plane is in the free space. The present effort uses Richmond's results in reference 1 to obtain the far-zone field when the ground plane is above flat Earth. Numerical evaluation of the far zone field is achieved with Richmond's computer program RICHMOND4.

Consideration is limited to the far-zone field in the free-space region, with ionospheric effects excluded. When the observer approaches the air-Earth interface, the total far-zone field will include a small contribution from the "surface wave," but this term is not considered here. Instead of a null on the radio horizon, the surface wave will contribute a far-zone evanescent field that is small compared to the far-zone field in the direction of peak directivity. However, the significant effect of the near-field surface wave in reducing radiation efficiency is included in the present analysis.

The radiation field from the surface magnetic current density (magnetic frill) of the coaxial line feed is included in the present analysis. Although the magnetic frill is the excitation source for the current on the monopole element and the disk ground plane, its contribution to the far-zone field may usually be neglected (as was done in references 11 and 12) unless the monopole element is so short that the radiation resistance of the magnetic frill becomes comparable to that of the monopole element.

The theoretical model, numerical results, validation of numerical results, and conclusions are given in sections 2, 3, 4, and 5, respectively. A more comprehensive treatment and review of monopole elements on circular ground planes in proximity to flat Earth is given in reference 13.

SECTION 2

THEORETICAL MODEL

2.1 METHODOLOGY

The antenna geometry consists of a vertical monopole element (length h and radius b), on an infinitely thin disk ground plane of radius a at a height z_o above flat Earth (see figure 1). The Earth, with a dielectric constant ε_r, conductivity σ (S/m) at a radian frequency ω (rad/s), and free-space wavelength λ (m), has a complex relative permittivity $\varepsilon^*/\varepsilon_o = \varepsilon_r (1 - j \tan \delta)$ where $\tan \delta$ = loss tangent = $\sigma/(\omega \, \varepsilon_r \, \varepsilon_o) = (\lambda\sigma/2\pi \, \varepsilon_r) \, (\mu_o/\varepsilon_o)^{1/2} \approx 60 \, \lambda\sigma / \varepsilon_r$. The monopole element and disk are assumed to have infinite conductivity. The location of an arbitrary far-zone observation point P is designated by spherical coordinates (ρ, θ, ϕ) with original O at the air-Earth interface below the monopole element.

The feed for the monopole antenna is a coaxial line with its inner conductor connected through a hole of radius b_1 in the ground plane to the vertical monopole element and its outer conductor connected by means of a flange to the ground plane. The inner conductor's diameter is equal to the monopole element's diameter $2b$ and the outer conductor's diameter is equal to the ground-plane hole diameter $2b_1$. The current on the outside of the coaxial-line feed is assumed to be zero because of the attenuation by lossy ferrite toroids along the exterior of the coaxial-line feed (see section 2.4 of reference 12). The coaxial line feed excitation may be replaced by an equivalent surface magnetic current density (magnetic frill) M_ϕ given by equation (23) of section 2.4.

The magnetic frill excitation gives rise to a monopole element current distribution $I_z(z)$ along the z axis of the monopole element and a disk current density distribution $J_\rho(\rho)$ in the radial direction ρ in the plane of the disk. The current density $J_\rho(\rho)$ is the net current density on the top and bottom of the disk (see equation 2.4.1 of reference 12). The method-of-moment solution for the distributions $I_z(z)$ and $J_\rho(\rho)$ is described in reference 1. These

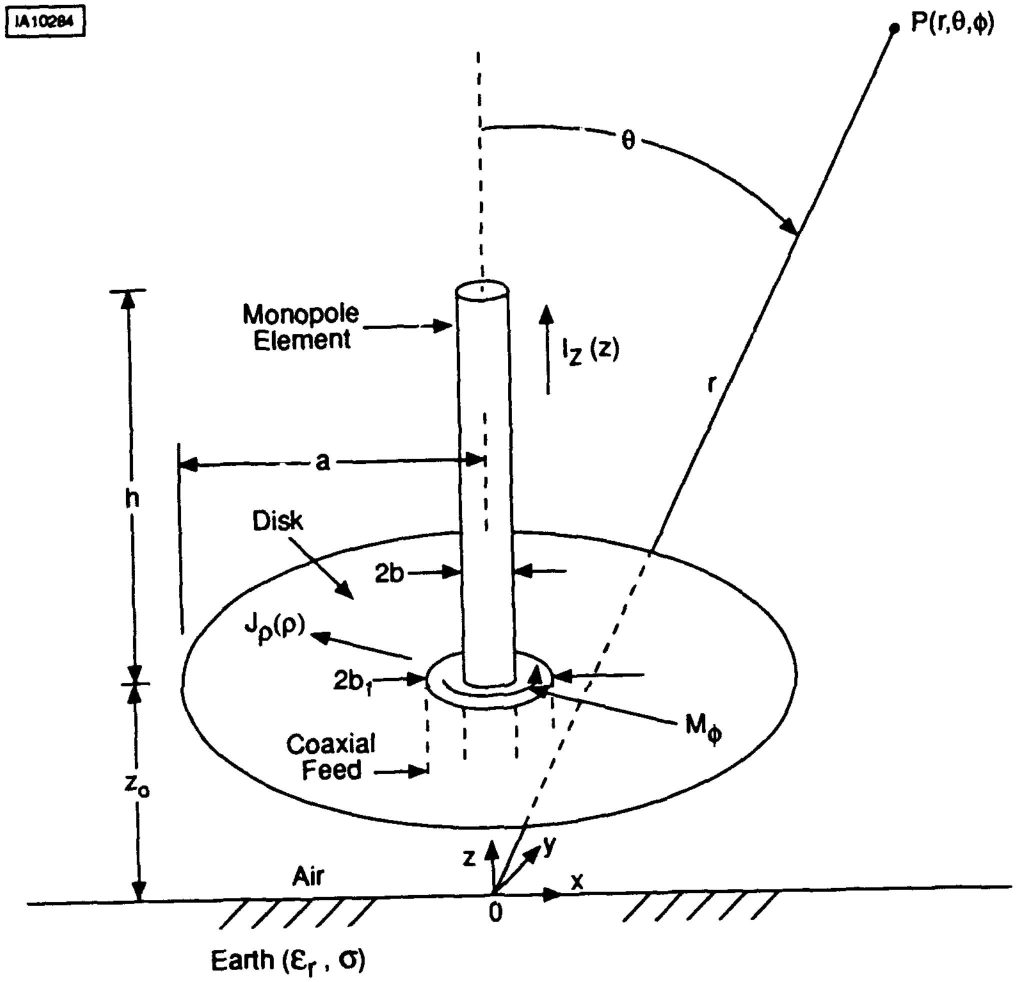


Figure 1. Monopole Element on Disk Ground Plane Above Flat Earth

currents are used to determine the far-zone field at $P(r,\theta,\phi)$. The distribution $I_z(z)$ and $J_\rho(\rho)$ include the contribution of the surface wave. The theory which follows is based on a report by Richmond (see Acknowledgments).

Wherever possible the notation will agree with that of references 1 and 10, except for the following: the disk radius is denoted by a (instead of c); the monopole element radius is noted by b (instead of a); the disk hole radius, equal to the radius of coaxial line outer conductor, is denoted by b_1 (instead of b); the Earth complex permittivity is denoted by ε^* (instead of ε_2); the free-space wave impedance is denoted by η_0 (instead of η); and the origin of the z axis is on the Earth's surface rather than on the disk ground plane. These changes in notation are made to conform with the notation for the numerical results of section 3. The time dependence is $exp(j\omega t)$. The parameters of free space are denoted by (μ_o, ε_o), $\kappa = \omega\sqrt{\mu_o \varepsilon_o}$, and $\eta_o = \sqrt{\mu_o / \varepsilon_o}$.

The far-zone electric field intensity of the monopole/disk antenna may be regarded as the sum of the field E^J radiated from the electric currents and the field E^M radiated from the magnetic frill current at the antenna terminals. To calculate these fields we consider the electric current density J, the magnetic current density M radiating in free space, and the field reflected from the air-Earth interface. In these calculations, the perfectly conducting antenna structure is removed and replaced with the equivalent currents J and M.

One successful but tedious approach to the far-zone fields starts with the rigorous expressions in terms of Sommerfeld integrals. This formulation is interpreted as a plane-wave expansion that includes a finite spectrum of uniform "space" waves plus an infinite spectrum of evanescent plane "surface" waves. Since the evanescent waves attenuate approximately exponentially with their height above the Earth, and since their peak amplitude relative to that of the space waves approach zero with increasing distance into the far-field, they are deleted in the far-zone field derivations. Finally, the method of stationary phase is applied to evaluate the remaining integrals asymptotically as the observation point recedes to infinity.

The same far-zone field expressions can be derived more readily via Carson's reciprocity theorem [22,23] as follows. The coordinate origin is on the air-Earth interface, the disk in the plane $z = z_o$, and the wire monopole extends from z_o to $z_o + h$ on the z axis. A point in the far-zone region is specified by the spherical coordinates (r, θ, ϕ) where r denotes the radial distance from the origin and the angle θ is measured from the z axis.. The goal is to determine the far-zone field $\boldsymbol{E}(r,\theta,\phi)$ in the free-space region. To accomplish this, we let $\left(\boldsymbol{E}^t,\boldsymbol{H}^t\right)$ denote the total field in the vicinity of the origin produced by an infinitesimal electric test dipole located at the far-zone point (r,θ,ϕ) in the plane $\phi = 0$. If the dipole is oriented in the $\hat{\boldsymbol{\theta}}$ direction and has a dipole moment p, its field in the vicinity (x, y, z) of the origin is

$$\boldsymbol{E}_o^t(x,y,z) = \frac{-\hat{\boldsymbol{\theta}} jk\eta_o p\, exp(-jkR')}{4\pi r} \tag{1}$$

$$R' = r - (x \sin\theta + z \cos\theta) \tag{2}$$

With this field incident on the air-Earth interface, the total magnetic field above the Earth in the vicinity of the origin is

$$\boldsymbol{H}^t(x,y,z) = \frac{\hat{y} jkp\, exp(-jkr)\, exp(jkx \sin\theta)}{4\pi r}$$

$$\cdot\left[exp(jkz\cos\theta) + \Re\, exp(-jkz\cos\theta)\right] \tag{3}$$

where $\Re$ denotes the plane-wave Fresnel reflection coefficient at the air-Earth interface.

The Earth conductivity, permeability, and dielectric constant are denoted by (σ, μ_2, ε_r). We let $\mu_2 = \mu_o$, in which case the reflection coefficient $\Re$ (for parallel polarization) is given by

$$\Re = \Re_{||} = \frac{(\varepsilon^* / \varepsilon_o)\cos\theta - \sqrt{(\varepsilon^* / \varepsilon_o) - \sin^2\theta}}{(\varepsilon^* / \varepsilon_o)\cos\theta + \sqrt{(\varepsilon^* / \varepsilon_o) - \sin^2\theta}} \tag{4}$$

The complex relative permittivity of the Earth is

$$(\varepsilon^* / \varepsilon_o) = \varepsilon_r - j\sigma / (\omega \varepsilon_o) \tag{5}$$

From equation (3) and Maxwell's curl equations, the total electric field intensity above the Earth in the vicinity of the origin is

$$E_x^t = \frac{-jk\eta_o p \exp(-jkr) \exp(jkx \sin\theta) \cos\theta}{4\pi r}$$

$$\cdot \left[\exp(jkz \cos\theta) - \Re \exp(-jkz \cos\theta)\right] \tag{6}$$

$$E_y^t = 0 \tag{7}$$

$$E_z^t = \frac{-jk\eta_o p \exp(-jkr) \exp(jkx \sin\theta) \cos\theta}{4\pi r}$$

$$\cdot \left[\exp(jkz \cos\theta) + \Re \exp(-jkz \cos\theta)\right] \tag{8}$$

Carson's reciprocity theorem states that

$$\int \boldsymbol{I}^t \cdot \boldsymbol{E} \, dl = \iint \left(\boldsymbol{J} \cdot \boldsymbol{E}^t - \boldsymbol{M} \cdot \boldsymbol{H}^t \right) ds \tag{9}$$

where $\boldsymbol{E}$ is the total far zone field at (r, θ, ϕ).

On the right-hand side of this equation, J and M denote surface-current densities on the perfectly conducting monopole/disk antenna and the integration extends over the surface of the monopole/disk antenna. On the left side the integration extends over the infinitesimal test dipole and is readily evaluated to be $pE_\theta(r,\theta,\phi)$, so the reciprocity theorem reduces to

$$E_\theta(r,\theta,\phi) = (1/p)\iint\left(J\cdot E^t - M\cdot H^t\right)ds \tag{10}$$

where p is the dipole moment.

The magnetic-frill current M is given. Upon completion of the moment-method analysis, the electric current density J is known on the vertical wire monopole and the horizontal conducting disk. The test dipole fields are given above, so equation (10) contains no unknown quantities. Thus, evaluation of the far-zone field $E_\theta(r, \theta, \phi)$ of the monopole/disk antenna is now simply a matter of performing the integrations in equation (10). If we start with a $\hat{\phi}$-oriented test dipole, a similar analysis shows that the resulting far-zone ϕ component of the monopole/disk antenna is $E_\phi(r, \theta, \phi) = 0$.

2.2. THE FIELD FROM THE MONOPOLE ELEMENT

In far-zone field calculations for the vertical wire monopole, the tubular surface current density J can be replaced with a filamentary line source $I(z)$ on the z axis. From equations (8) and (10), the far-zone field from the vertical wire is given by

$$E_\theta^w(r,\theta,\phi) = \frac{jk\eta_o \exp(-jkr)\sin\theta}{4\pi r}$$

$$\cdot\int_{z_o}^{z_o+h} I(z)\left[\exp(jkz\cos\theta) + \Re\exp(-jkz\cos\theta)\right]dz \tag{11}$$

The vertical wire monopole is divided into L segments with length $d' = L/h$. A typical segment (segment ℓ) extends from z_1^ℓ to z_2^ℓ on the z-axis, with the following current distribution:

$$I^\ell(z) = \frac{I_1^\ell \sin k\left(z_2^\ell - z\right) + I_2^\ell \sin k\left(z - z_1^\ell\right)}{\sin kd'} \tag{12}$$

The current entering the segment at the bottom is $I_2^\ell = I\left(z_1^\ell\right)$, and the current leaving the segment at the top is $I_2^\ell = I\left(z_2^\ell\right)$.

From equations (11) and (12), the far-zone field of the wire monopole is given by

$$\begin{aligned} E_\theta^w(r,\theta,\phi) = C \sum_{\ell=1}^{L} I_1^\ell \Big\{ & exp\left(jkz_2^\ell \cos\theta\right) - A\ exp\left(jkz_1^\ell \cos\theta\right) \\ & +\Re \left[exp\left(-jkz_2^\ell \cos\theta\right) - B\, exp\left(-jkz_1^\ell \cos\theta\right)\right]\Big\} \\ & +C \sum_{\ell=1}^{L} I_2^\ell \Big\{ exp\left(jkz_1^\ell \cos\theta\right) - B\, exp\left(jkz_2^\ell \cos\theta\right) \\ & +\Re \left[exp\left(-jkz_1^\ell \cos\theta\right) - A\, exp\left(-jkz_2^\ell \cos\theta\right)\right]\Big\} \end{aligned} \tag{13}$$

where

$$A = \cos(kd) + j \cos\theta \sin(kd) \tag{14}$$

$$B = \cos(kd) - j \cos\theta \sin(kd) \tag{15}$$

$$C = \frac{j\eta_o \, exp(-jkr)}{4\pi r \; sin(kd) \; sin\,\theta} \tag{16}$$

On the lowest wire segment ($\ell = 1$), the current at the bottom is $I_1^\ell = I_1$. On the highest segment ($\ell = L$) the endpoint currents are $I_1^\ell = I_N$ and $I_2^\ell = 0$, where N denotes the number of equations and the number of unknowns in the moment-method solution for the monopole/disk antenna.

2.3 THE FIELD FROM THE DISK GROUND PLANE

The electric current density $J_\rho(\rho)$ on the perfectly conducting circular disk is radially directed and independent of the azimuthal angle ϕ. The disk lies in the plane $z = z_O$. If (ρ', ϕ', z_o) denotes the cylindrical coordinates of a source point on the disk, the far-zone disk field is obtained from equations (6), (7), and (10) as follows:

$$E_\theta^d(r,\theta,\phi) = \frac{-jk\eta_o \, exp(-jkr) \, cos\,\theta}{4\pi r}\left[exp(jkz_o \, cos\,\theta) - \Re \; exp(-jkz_o \, cos\,\theta)\right] \tag{17}$$

$$\cdot \int_b^a \int_{-\pi}^{\pi} J_\rho(\rho') cos\,\phi'' \, exp(jk\rho' \, cos\,\phi'' \, sin\,\theta) \rho' d\phi'' d\rho'$$

$$\phi'' = \phi' - \phi \tag{18}$$

Since the disk current density J_ρ is independent of ϕ', one integration can be evaluated as follows:

$$\int_{-\pi}^{\pi} cos\,\phi \; exp(jx \; cos\,\phi) \, d\phi = 2\pi j \; J_1(x) \tag{19}$$

where $J_1(x)$ denotes the Bessel function. Beginning at this point, it is convenient to let ρ (instead of ρ') denote the radial coordinate of a source point on the disk. From equations (17) and (19), the far-zone field of the circular disk is

$$E_\phi^d = 0.5\, k\eta_o \left[exp(-jkr)/r\right] cos\,\theta$$

$$\cdot\left[exp(jkz_o\, cos\,\theta) - \Re\, exp(-jkz_o\, cos\,\theta)\right] \int_b^a \rho J_\rho(\rho)\, J_1(k\rho\, sin\,\theta)\, d\rho \qquad (20)$$

The perfectly conducting circular disk is divided into M concentric annual zones. A typical zone (zone m) has an inner radius ρ_1^m, an outer radius ρ_2^m, and a width $d = \rho_2^m - \rho_1^m = (a-b)/M$. Let I_1^m denote the electric current entering the zone at ρ_I^m, and I_2^m the current leaving at ρ_2^m. Then the electric surface current density on this zone is

$$J_\rho^m(\rho) = \frac{I_1^m\, sin\, k\left(\rho_2^m - \rho\right) + I_2^m\, sin\, k\left(\rho - \rho_1^m\right)}{2\pi\rho\, sin\, kd} \qquad (21)$$

From equations (20) and (21), the far-zone field of the circular disk is given by

$$E_\theta^d(r,\theta,\phi) = \frac{\eta_o\, exp(-jkr)\, cos\,\theta}{4\pi r\; sin(kd)}\left[exp(jkz_o\, cos\,\theta) - \Re\, exp(-jkz_o\, cos\,\theta)\right]$$

$$\cdot\sum_{m=1}^{M}\int_{k\rho_1^m}^{k\rho_2^m}\left[I_1^m\, sin\, k\left(\rho_2^m - \rho\right) + I_2^m\, sin\, k\left(\rho - \rho_1^m\right)\right] J_1(k\rho\; sin\,\theta)\, d(k\rho) \qquad (22)$$

On the first zone (m = 1), the endpoint currents are $I_1^m = -I_1$ and $I_2^m = I_2$. On the last zone ($m = M$), the endpoint currents are $I_1^m = I_M$ and $I_2^m = 0$. Numerical integration techniques are required in evaluating this expression.

2.4 THE FIELD FROM THE MAGNETIC FRILL

The perfectly conducting circular disk and the coaxial-fed monopole are replaced (via Schelkunoff's equivalence principle) with equivalent electric and magnetic surface currents radiating in free space over the flat Earth. The equivalent magnetic surface-current density, derived in section 2.4 of reference 12, is given by:

$$M_\phi = \begin{cases} -V/[\rho \; \ell n(b_1/\mathrm{b})], & b \le \rho \le b_1 \\ 0, & \rho \text{ elsewhere} \end{cases} \tag{23}$$

This "magnetic frill," located at $z = z_o$, is centered on the z-axis and has inner and outer radii of b and b_1, respectively. The antenna is considered to be transmitting, with a voltage generator (of V peak volts) at the terminals and the coaxial outer conductor at zero potential. The free-space field of the magnetic frill is analyzed by Tsai [14,15]. From equations (3) and (10), the far-zone field of the frill is given by

$$E_\theta^M(r,\theta,\phi) = \frac{-jk\,exp(-jkr)}{4\pi r}\left[exp(jkz_o \cos\theta) + \Re\, exp(-jkz_o \cos\theta)\right]$$

$$\cdot \int_b^{b_1} \int_{-\pi}^{\pi} M_\phi(\rho')\cos\phi''\, exp(jk\rho' \cos\phi'' \sin\theta)\rho' d\phi'' d\rho' \tag{24}$$

Since the magnetic current density is independent of ϕ', one integration can be performed with the aid of equations (19) and (23) to obtain

$$E_\theta^M = \frac{-kV\,exp(-jkr)}{2r\;\ln(b_1/b)}\left[exp(jkz_o \cos\theta) + \Re\, exp(-jkz_o \cos\theta)\right] \cdot \int_b^{b_1} J_1\,(k\rho \sin\theta)\, d\rho \tag{25}$$

The final integration is performed as follows:

$$\int J_1(\beta x)\, dx = -J_o \beta x/\beta \tag{26}$$

Thus, the field of the magnetic frill is given by

$$E_\theta^M = \frac{V\, exp(-jkr)}{2r\, \ell n(b_1/b)}\left[exp(jkz_o \cos\theta) + \Re\, exp(-jkz_o \cos\theta)\right]$$

$$\cdot\left[J_o(kb \sin\theta) - J_o(ka \sin\theta)\right]/\sin\theta \tag{27}$$

This expression can be simplified with the following:

$$J_o(x) \approx 1 - x^2/4, \quad x << 1 \tag{28}$$

From equations (27) and (28), the far-zone field of the magnetic frill is given by

$$E_\theta^m(r,\theta,\phi) = \frac{k^2 V\left(b^2 - b_1^2\right) exp(-jkr)}{8r\ \ell n(b_1/b)}$$

$$\cdot\left[exp(jkz_o \cos\theta) + \Re\, exp(-jkz_o \cos\theta)\right] \sin\theta, \quad kb_1 << 1 \tag{29}$$

2.5 THE TOTAL FAR-ZONE FIELD

The total far-zone field $E_\theta(r,\theta,\phi)$ defined by equation (10) in the free-space region is the sum of the fields from the monopole element, the disk ground plane, and the magnetic frill. Accordingly, the total far-zone field in the free-space(air) region is given by

$$E_\theta(r,\theta,\phi) = E_\theta(r,\theta) = E_\theta^w(r,\theta) + E_\theta^d(r,\theta) + E_\theta^M(r,\theta) \tag{30}$$

where E_θ^w, E_θ^d, E_θ^M are given by equations (13), (22), and (29), respectively. The fields E_θ, E_θ^w, E_θ^d, and E_θ^M are uniform with azimuthal angle ϕ because of the azimuthal symmetry of the antenna geometry in figure 1.

Consider now the cases where the Earth medium either is lossy $(\sigma > 0)$ or is free space $(\sigma = 0,\ \varepsilon_r = 1)$. The total far-zone radiated power P_r is given by

$$P_r = \begin{cases} (\pi/\eta_o) \Big/ \int_o^{\pi/2} |E_\theta(r,\theta)|^2 r^2 \sin\theta \; d\theta, & \sigma > 0 \\ (\pi/\eta_o) \Big/ \int_o^{\pi} |E_\theta(r,\theta)|^2 r^2 \sin\theta \; d\theta; & \sigma = 0 \end{cases} \tag{31}$$

where $E_\theta(r,\theta)$ = the far-zone field (in the free-space region) given by equation (30).

$\eta_o = (\mu_o/\varepsilon_o)^{1/2}$ = free space wave impedance (ohms)

For the case of $\sigma > 0$, the integrand in equation (31) is integrated over only the hemisphere above the Earth because the field in lossy Earth, relative to that in free space, approaches zero at large radial distances r.

The antenna directivity $d(\theta)$ expressed as a numeric is given by

$$d(\theta) = 2\pi\, r^2 |E_\theta(r,\theta)| / (\eta_o P_r) \tag{32}$$

The antenna directivity $D(\theta)$, expressed in decibels, is given by

$$D(\theta) = 10 \log_{10} d(\theta) \quad (dB) \tag{33}$$

The input power P_{in} to the monopole element is given by

$$P_{in} = (1/2) \; Re[V(0) I^*(0)] \tag{34}$$

where $V(0)$ = Peak input voltage (volts). The input voltage $V(0)$ is usually set equal to 1 volt in the moment-method analysis.

$I^*(0)$ = Conjugate of the peak input current $I(0)$ at the base of the monopole element. This current is solved for by the moment-method analysis in reference 1.

The input impedance Z_{in} is given by

$$Z_{in} = R_{in} + j\,X_{in} = V(0)/I(0) \tag{35}$$

where R_{in} and X_{in} are the input resistance and reactance, respectively.

The antenna radiation resistance R_{rad} is defined as

$$R_{rad} = 2P_r / |I(0)|^2 \tag{36}$$

The antenna radiation efficiency η is defined as

$$\eta = P_r / P_{in} = \left[1 + (R_{rad}/R_{in})\right]^{-1} \tag{37}$$

For the case of free-space ($\sigma = 0$, $\varepsilon_r = 1$), the radiation efficiency is equal to unity because the monopole element and the disk ground plane conductivities are assumed to be infinite.

SECTION 3

NUMERICAL RESULTS

Numerical evaluation of the far-zone field, directivity, radiation resistance, and radiation efficiency is executed by Richmond's computer program RICHMOND4 written in FORTRAN 77, with double precision for use on a DEC VAX computer. The program RICHMOND4 uses subroutines from Richmond's computer program RICHMOND3 that determines the current distributions on the monopole element and disk ground plane, as well as the input current $I(Z_0)$ and input impedance $Z = V/I(z_0)$. Brief descriptions, listings, and sample outputs by Richmond of programs RICHMOND3 and RICHMOND4 are given in appendices A and B, respectively. Programs RICHMOND3 and RICHMOND4 are extensions of programs RICHMD1 and RICHMD2, respectively, described in reference 12 for a monopole element on a disk ground plane in free space.

Examples of numerical results are presented here for a thin, quarter-wave monopole element on a small to moderately large disk ground plane resting on medium dry ground at 15 MHz in the high-frequency band ($b/\lambda = 10^{-6}$, $h/\lambda = 0.25$, $2\pi a/\lambda = 0$ to 8 wavenumbers, $z_0 = 0$, $\varepsilon_r = 15$, $\sigma = 0.001$ S/m, $\tan\delta = 60\lambda\sigma/\varepsilon_r = 0.08$). More extensive results, in the form of an atlas of computer plots, are presented in reference 21 as a function of Earth classification. The coaxial line feed ($b_1/b = 3.5$) has a negligible effect on the far-zone field and input current because its equivalent magnetic frill of outer diameter $2b_1/\lambda$ (= 7 x 10^{-6} wavelengths) has a radiation resistance that is small compared to that of the monopole element of length h/λ (= 0.25 wavelengths). In the numerical results, the monopole element was divided into four segments. The disk was segmented into equal-width annular zones, whose numbers varied from seven for ka = 0.025, 0.25, 0.50; 16 for ka = 0.75 through 5.25; 17 for ka = 5.5; 18 for ka = 5.75 and 6.00; 19 for ka = 6.25; 20 for ka = 6.5; 21 for ka = 6.75, 7.0; 22 for ka = 7.25; 23 for ka = 7.50; and 24 for ka = 7.75 and 8.0. Results are compared with those for a perfect ground plane ($\varepsilon_r = 1.0$, $\sigma = \infty$) and for an Earth permittivity equal to that of free space ($\varepsilon_r = 1.0$, $\sigma = 0$). The results for perfect ground, medium dry ground, and free space are identified in the following figures as Case 1, Case 5, and Case 11, respectively.

The elevation numeric directivity patterns for disk radii $2\pi a/\lambda = 0.025, 3.0, 4.0, 5.0$, and 6.5 wavenumbers are shown as polar plots on the same linear scale in figures 2 through 6, respectively. In the presence of Earth (Case 5), the directivity patterns are approximately independent of disk radius. The Earth softens the edge of the ground plane and minimizes changes in directive gain resulting from ground plane edge diffraction. The peak directivity (see figure 7) is within 0.5 dBi of that for a perfect ground plane. The direction of peak directivity (see figure 8) is approximately 30° above the horizon with variations of less than 4° for ground plane radii $0 \le 2\pi a/\lambda \le 8$ wavenumbers. The directivity at angles of incidence near the horizon (see figures 9 through 13) for $0 \le 2\pi a/\lambda \le 8$ wavenumbers has no improvement over that with no ground plane at all and, in fact, decreases periodically with increasing disk radius by as much as 1 dB. The directivity at angles of incidence of 82°, 84°, 86°, 88°, and 90° are approximately 4 dB, 5 dB, 7 dB, 13 dB, and ∞ dB, respectively, below the peak directivity for these disk radii.

The directivity on the horizon (see figure 13) is -∞ dB because of the space wave multipath null for Earth surface reflection at a grazing angle of 0°. In actuality, the field on the radio horizon is not zero because of the leaky evanescent surface wave that is generated in the air medium in proximity to the air-Earth interface [13]. The surface wave has an evanescent field in the air-medium only, but leaks energy into the Earth medium, not into the air medium. The amplitude of the space wave in the direction of peak directivity approaches zero with increasing distance into the far-zone.

The theoretical numeric directive gain of electrically short monopole elements on ground planes resting on lossy Earth may be approximated by an expression of the form [13]

$$d_r(\theta) = \begin{cases} A\cos^m\theta\sin^n\theta;\ 0 \le \theta \le \pi/2\ \ rad,\ m > 0, n > 1 \\ 0,\ -\pi/2 \le \theta < 0\ rad \end{cases} \tag{37}$$

f = 15 MHz

h/λ=0.25, b/λ=1.0 x 10^{-6}, z_0/λ=0
Case 1, Perfect Ground (ε_r=1.0, σ=∞)
Case 5, Medium Dry Ground (ε_r=15.0, σ=0.001 S/m)
Case 11, Free Space (ε_r=1.0, σ=0)

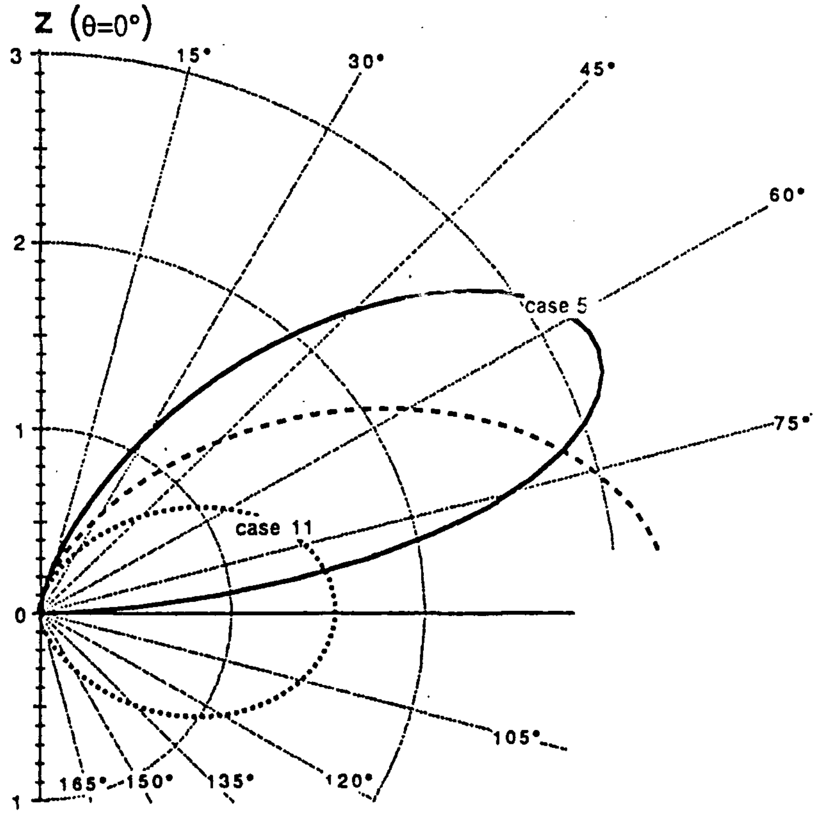


Figure 2. Numeric Directive Gain Polar Plot, $2\pi a/\lambda$ = 0.025

f = 15 MHz

h/λ=0.25, b/λ=1.0 x 10^{-6}, z_0/λ=0·
Case 1, Perfect Ground (ε_r=1.0, σ=∞)
Case 5, Medium Dry Ground
(ε_r=15.0, σ=0.001 S/m)
Case 11, Free Space (ε_r=1.0, σ=0)

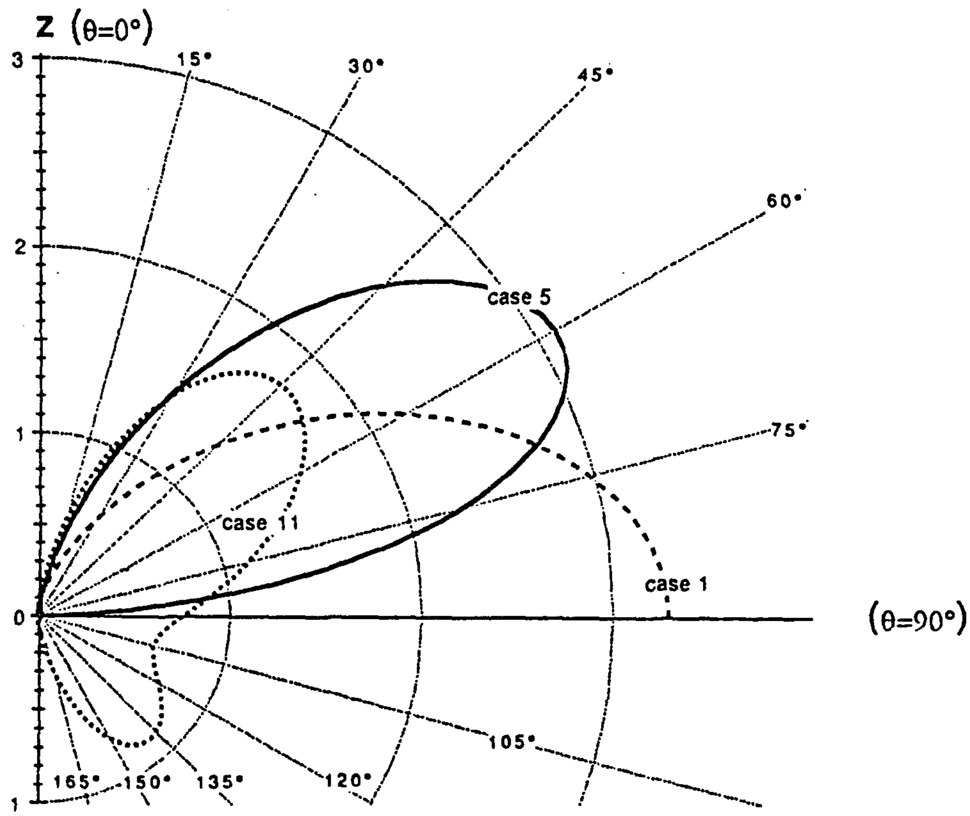


Figure 3. Numeric Directive Gain Polar Plot, $2\pi a/\lambda$ = 3.0

f = 15 MHz

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 5, Medium Dry Ground ($\varepsilon_r=15.0$, $\sigma=0.001$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

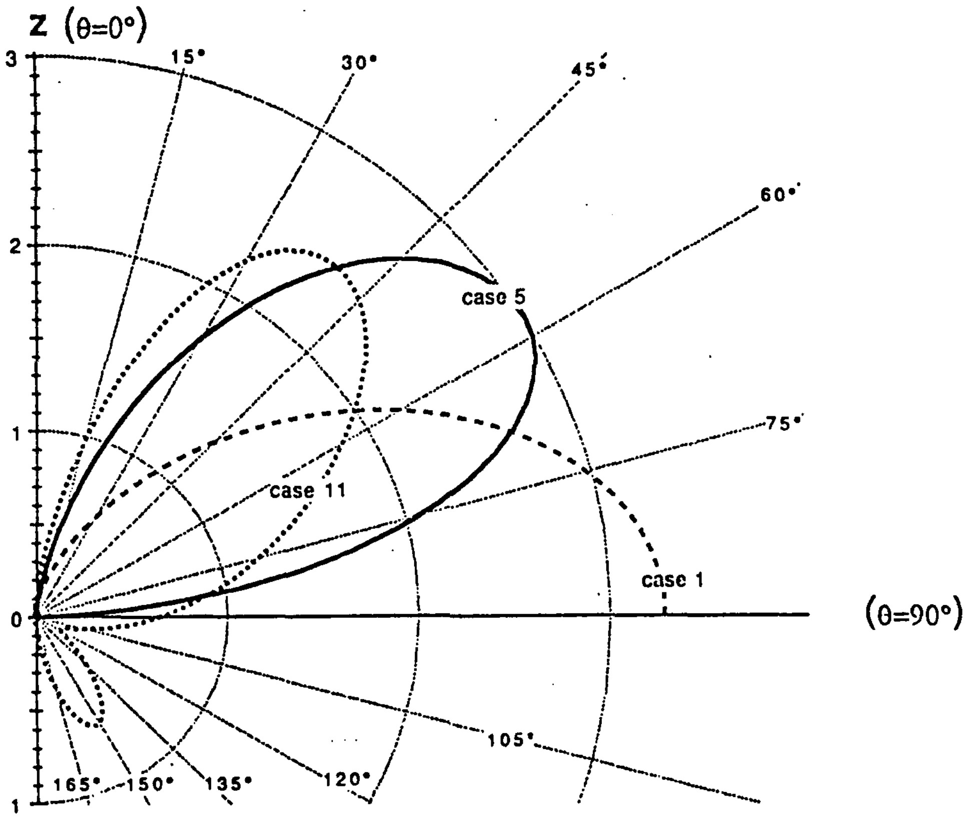


Figure 4. Numeric Directive Gain Polar Plot, $2\pi a/\lambda$ = 4.0

f = 15 MHz

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 5, Medium Dry Ground
($\varepsilon_r=15.0$, $\sigma=0.001$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

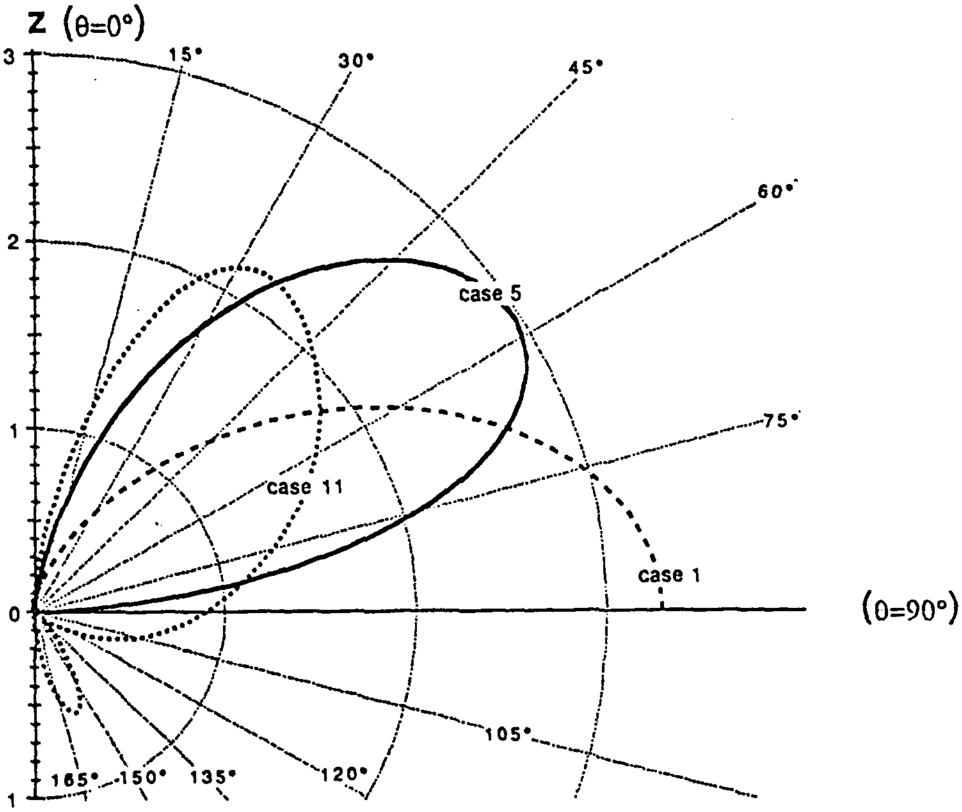


Figure 5. Numeric Directive Gain Polar Plot, $2\pi a/\lambda$ = 5.0

f = 15 MHz

h/λ=0.25, b/λ=1.0 x 10^{-6}, z_0/λ=0

Case 1, Perfect Ground (ε_r=1.0, σ=∞)

Case 5, Medium Dry Ground
(ε_r=15.0, σ=0.001 S/m)

Case 11, Free Space (ε_r=1.0, σ=0)

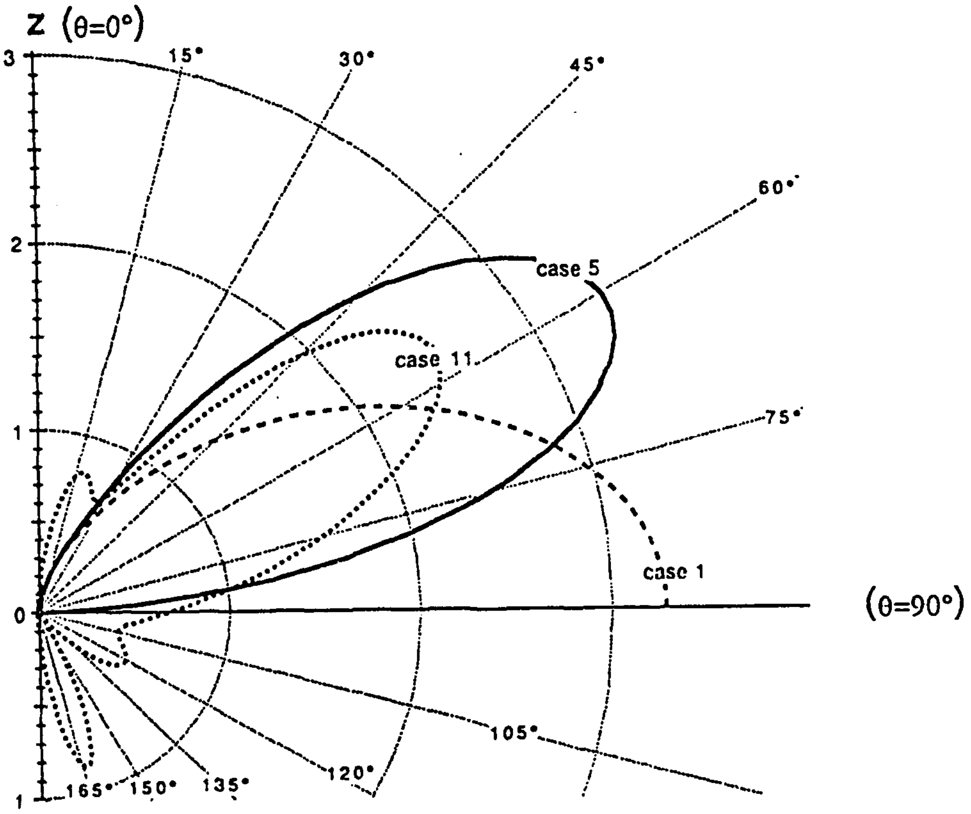


Figure 6. Numeric Directive Gain Polar Plot, $2\pi a/\lambda$ = 6.5

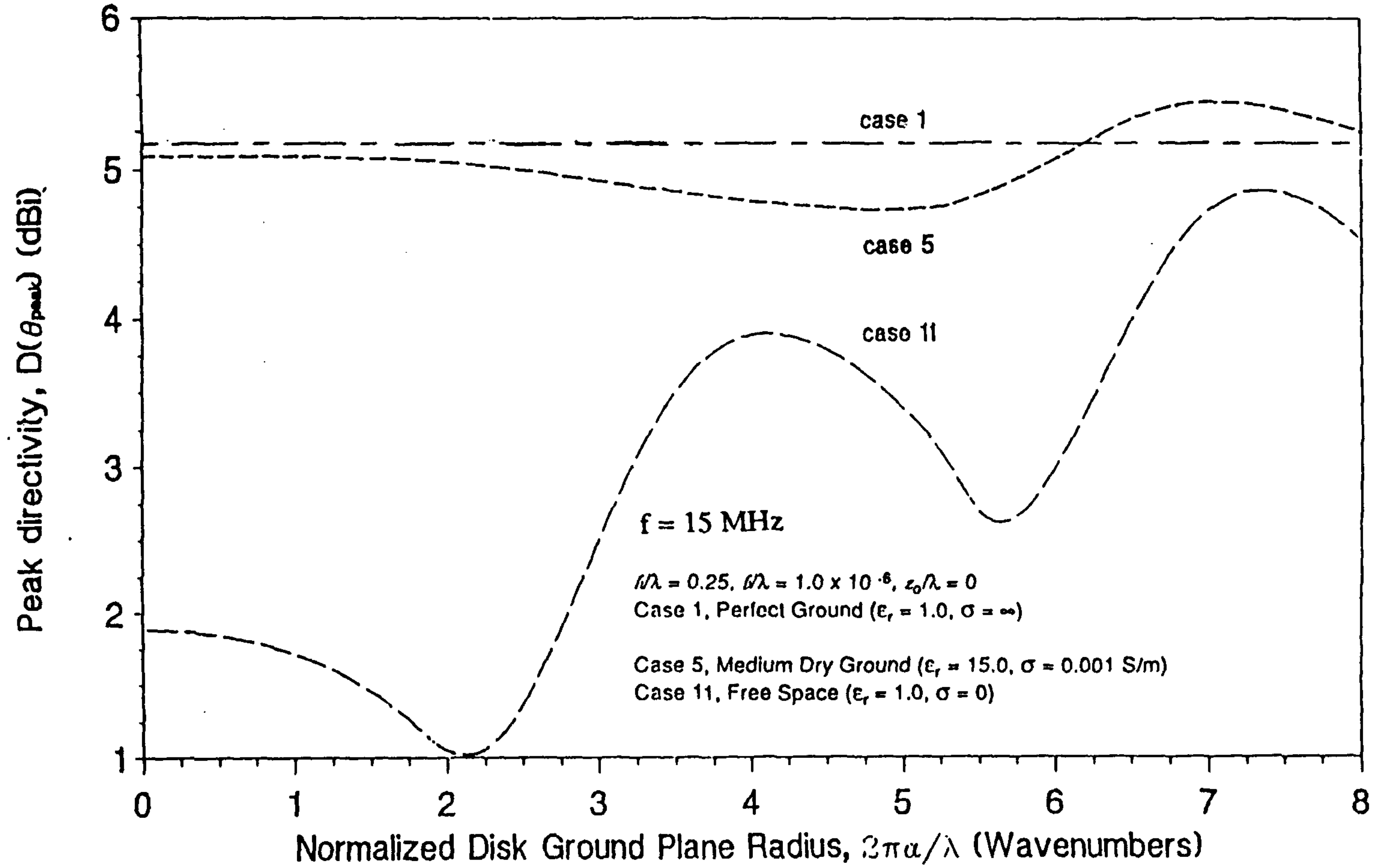


Figure 7. Peak Directivity

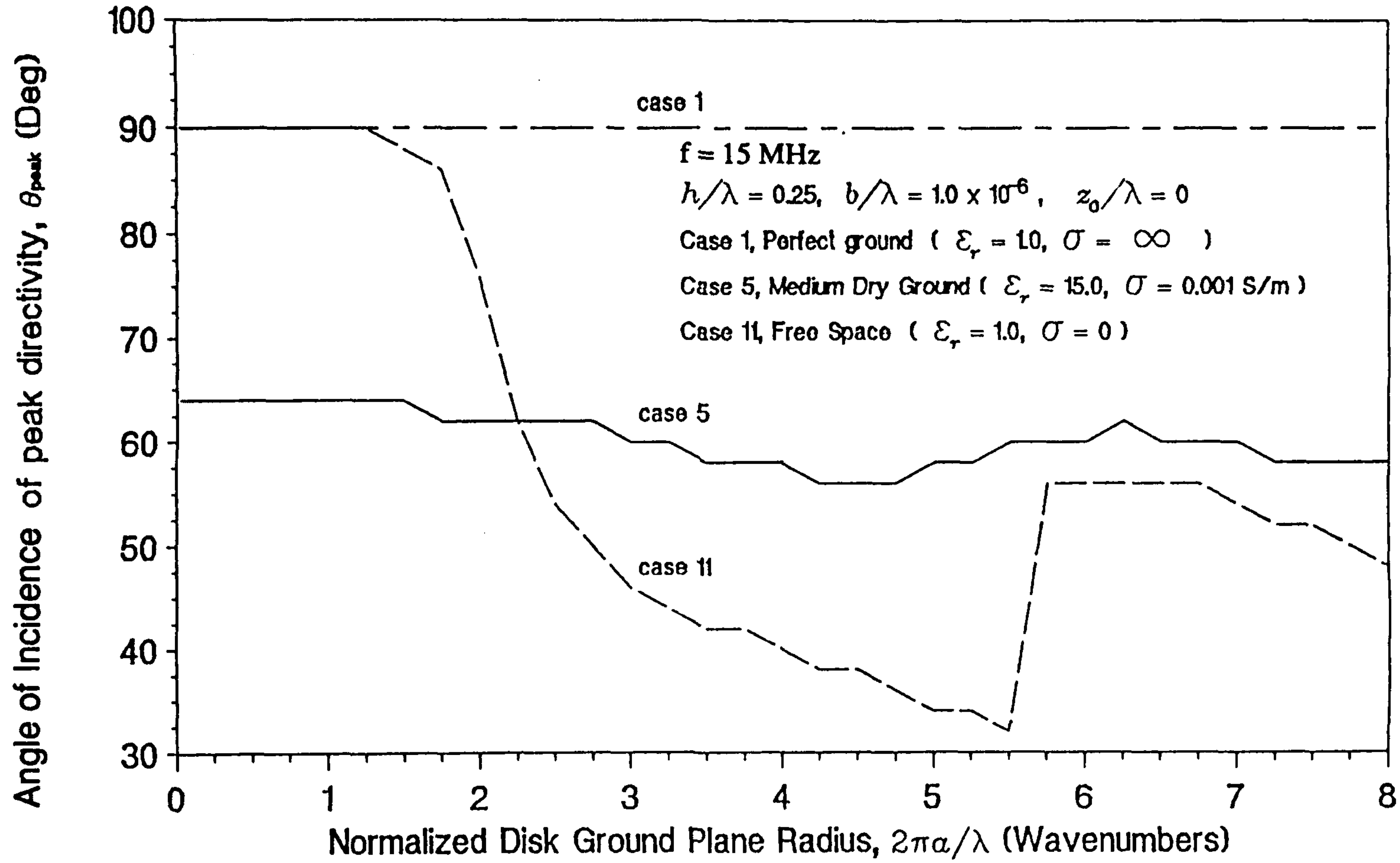


Figure 8. Angle of Incidence of Peak Directivity

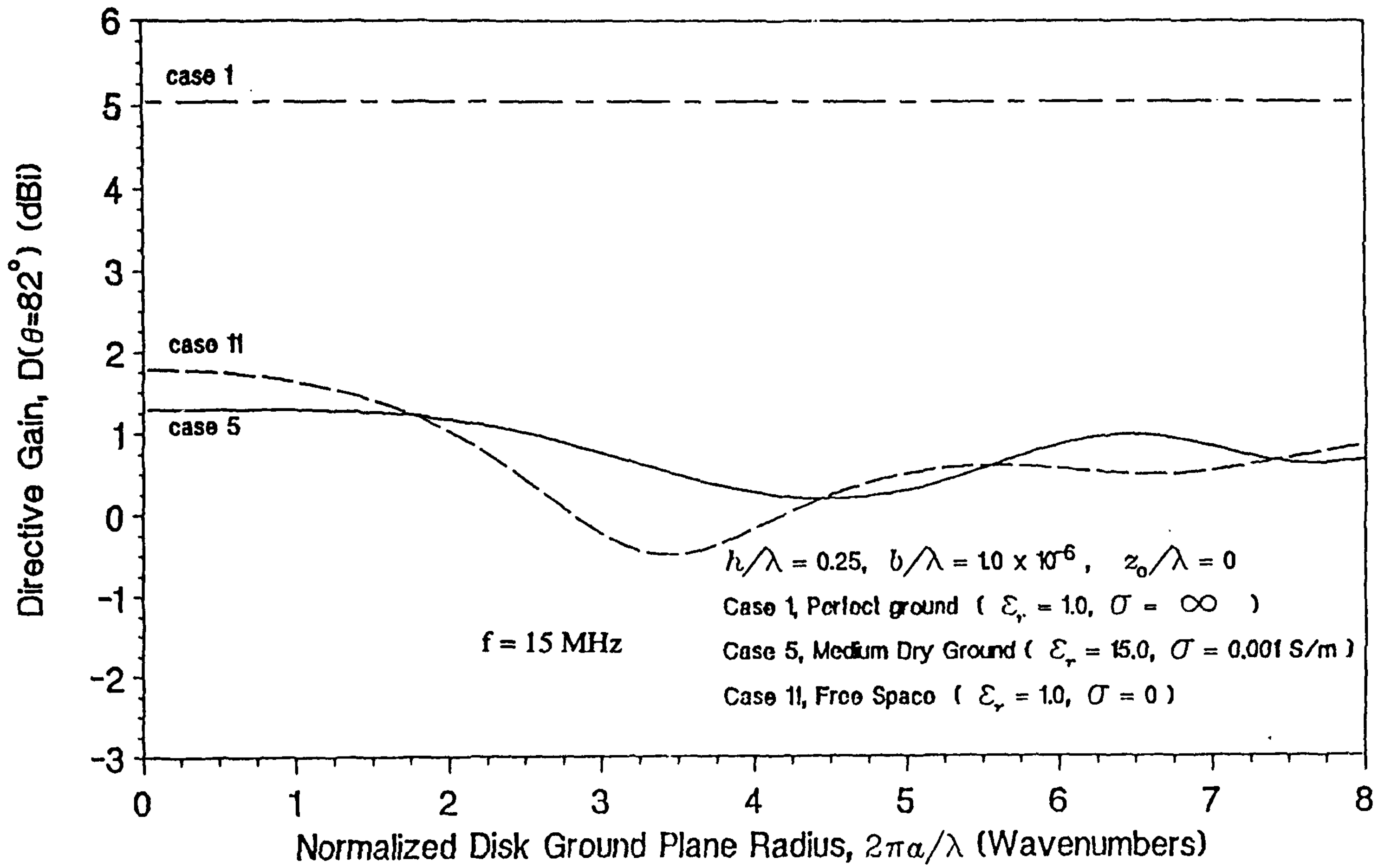


Figure 9. Directive Gain at Eight Degrees above the Horizon

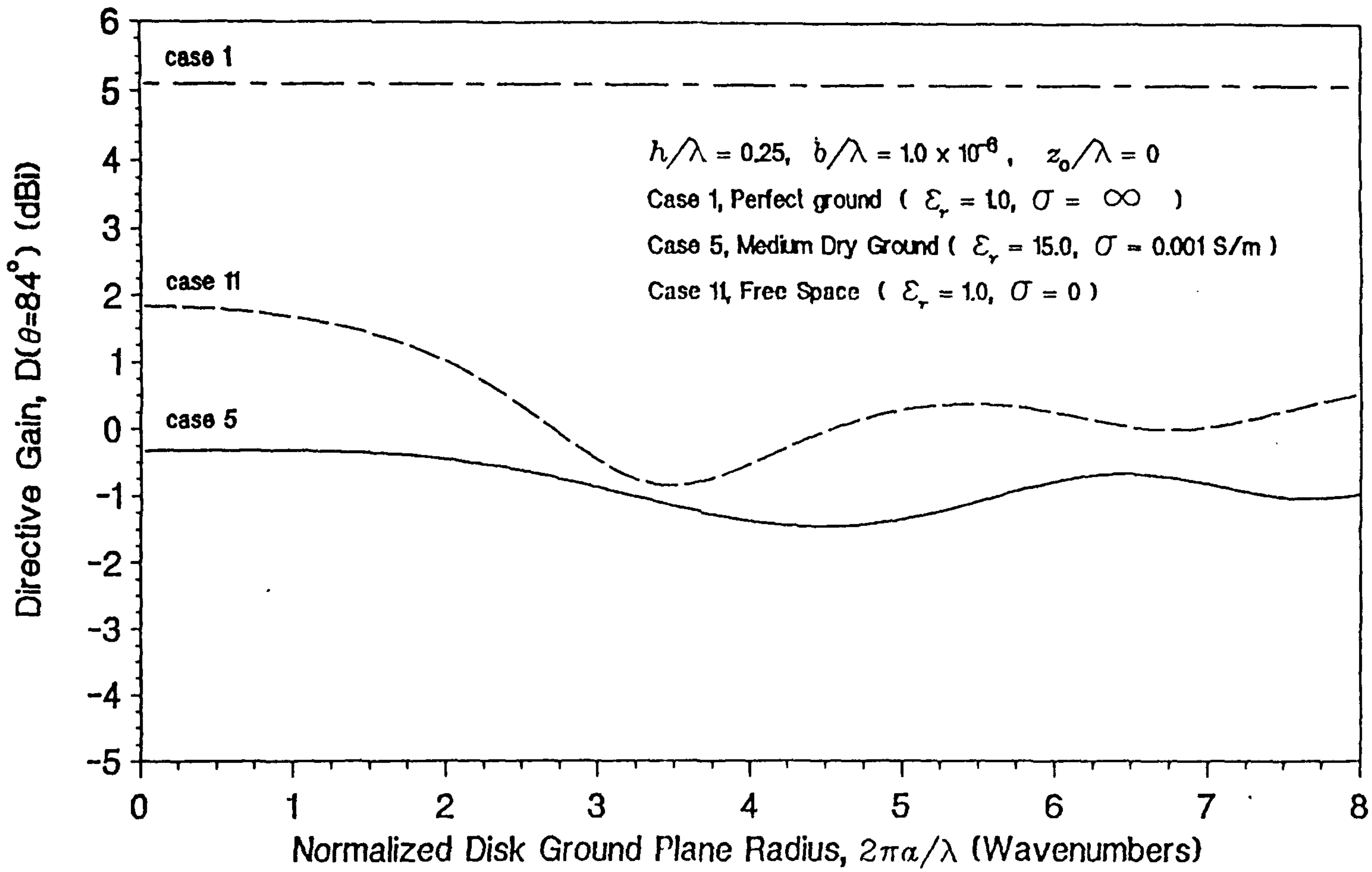


Figure 10. Directive Gain at Six Degrees above the Horizon

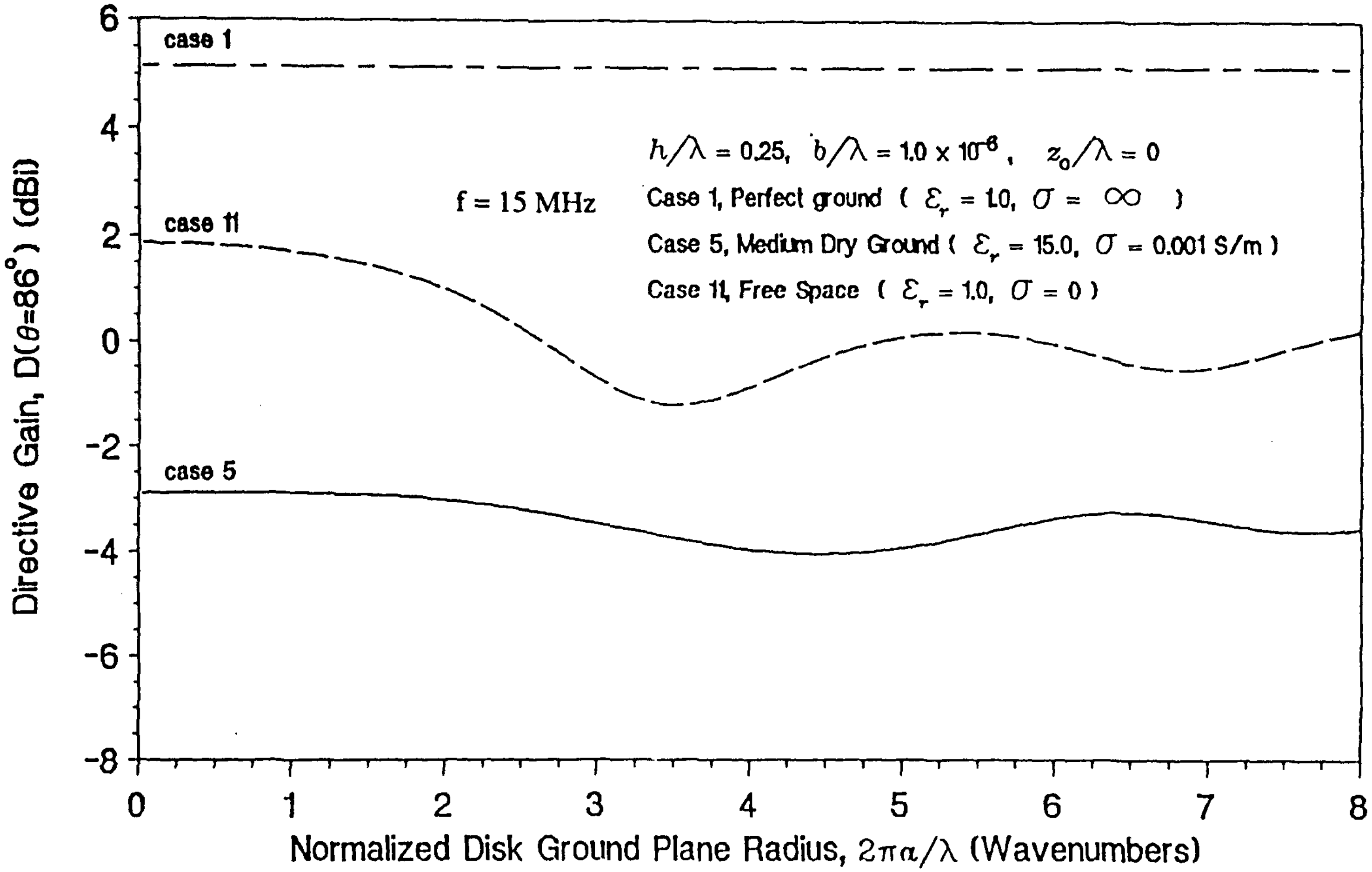


Figure 11. Directive Gain at Four Degrees above the Horizon

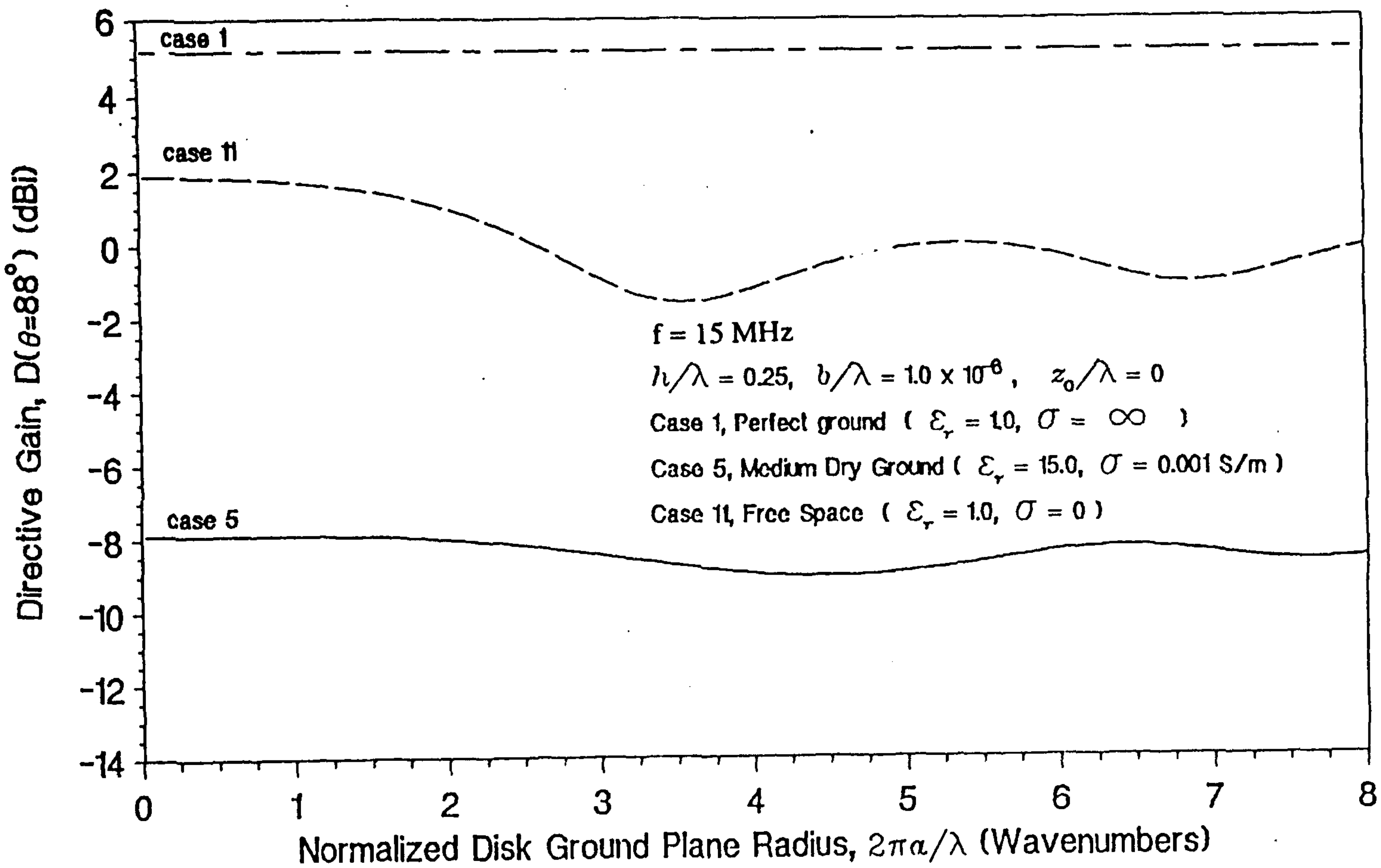


Figure 12. Directive Gain at Two Degrees above the Horizon

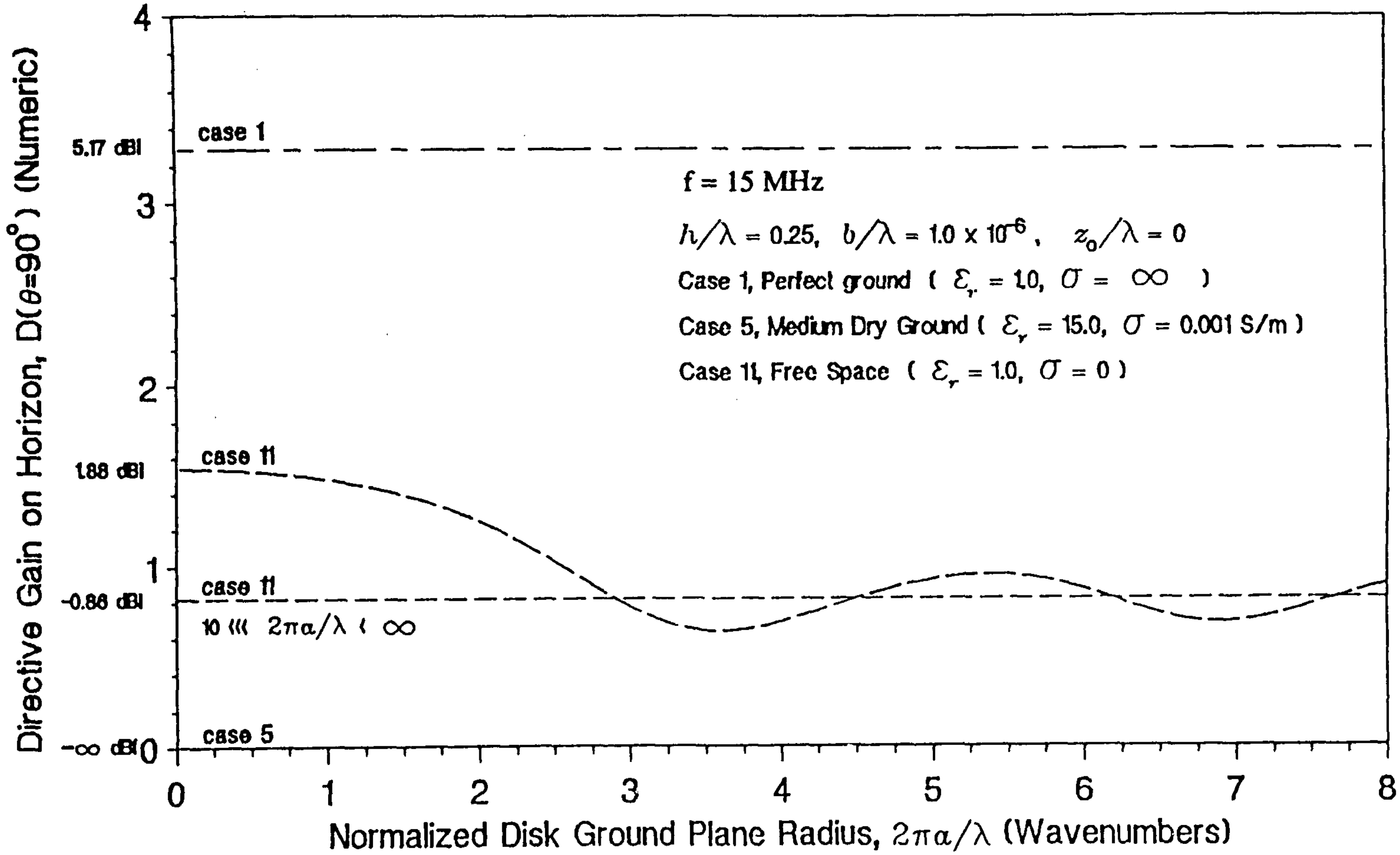


Figure 13. Directive Gain on the Horizon

The exponents m and n are chosen to yield a peak directivity in a desired direction, and a null at $\theta = 0$ and $\pi/2$ radians. The coefficient A is chosen to satisfy the condition

$$(1/4\pi)\int_0^{2\pi}\int_0^{\pi/2} d_r(\theta) sin\,\theta\, d\phi = 1$$

Accordingly,

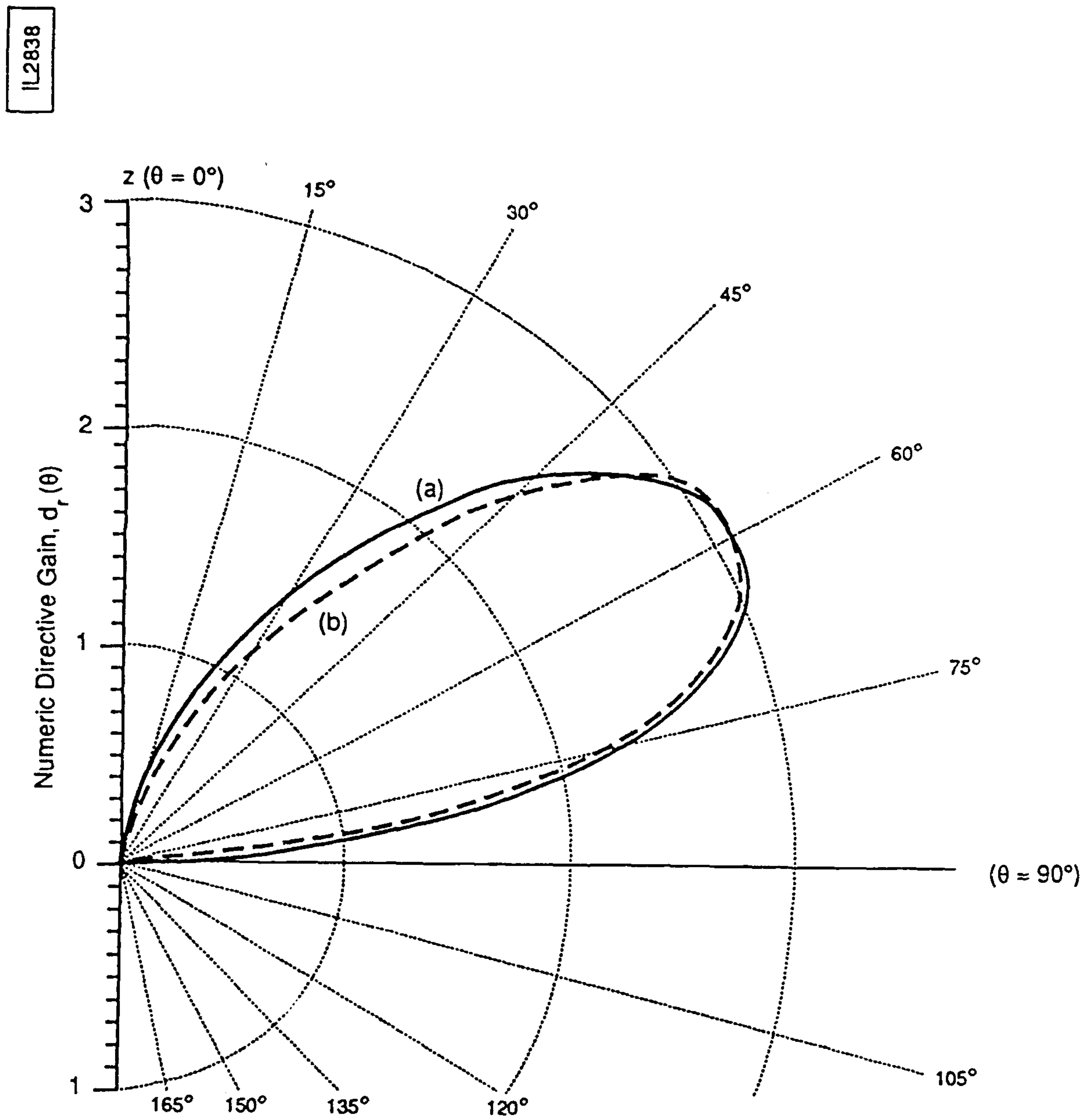
$$A = 2\Big/\int_0^{\pi/2} cos^m\theta\, sin^{n+1}\theta\, d\theta$$

The directivity of equation (37) has a null in the direction of zenith, and the horizon has a peak directivity comparable to that for a perfect ground plane.

An analytical expression that approximates the directivity obtained by numerical methods for medium dry ground and $2\pi a/\lambda = 3$ (see figure 14) is given by

$$d_r(\theta) = \begin{cases} 10 cos\,\theta\, sin^3\theta; & 0 \le \theta \le \pi/2 \;\; rad \\ 0, & -\pi/2 \le \theta < 0 \; rad \end{cases} \tag{38}$$

In the absence of Earth (Case 11), the directivity patterns (see figures 2 through 6) are strong functions of the disk radius because ground-plane edge diffraction is more pronounced. The peak directivity (see figure 7) varies from approximately 2 dBi to 5 dBi. The angle of incidence of peak directivity (see figure 8) varies from 0° to 32°. The large changes in angle of peak directivity at $2\pi a/\lambda = 5.5$ wavenumbers do not represent significant changes in peak directivity because of the broad 3-dB beamwidth of the directivity pattern. The jump in angle of peak directivity between $2\pi a/\lambda = 5.5$ and 5.75 wavenumbers corresponds to a change in beamshape (compare figures 5 and 6). The directivity on the horizon (see figure 13) varies from 1.88 dBi for $2\pi a/\lambda = 0$ wavenumbers to the asymptotic value of -0.88 dBi for large disk ground planes of finite radius.



$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$, $f = 15$ MHz
Case 5, Medium Dry Ground
($\epsilon_r = 15.0$, $\sigma = 0.001$ S/m)

Figure 14. Numeric Directive Gain of a Quarter-Wave Element on a Disk Ground Plane Resting on Medium Dry Ground, $2\pi a/\lambda = 3.0$ (a) Richmond's Method-of-Moments; (b) $10 \cos\theta \sin^3\theta$

The radiation resistance (see figure 15) increases aperiodically with increasing disk radius. The aperiodicity is more apparent in the absence of Earth because ground-plane edge diffraction is more pronounced. The radiation efficiency (see figure 16) increases monotonically with increasing disk radius in the presence of Earth: from 0.21 for $2\pi a/\lambda = 0$, to 0.69 for $2\pi a/\lambda = 8$, and to 1.0 for $2\pi a/\lambda = \infty$. In the absence of Earth, the radiation efficiency is equal to unity because the monopole element and disk are assumed to be of infinite conductivity. The reason why the radiation efficiency is so small for small disk ground planes in close proximity to Earth, regardless of whether the Earth is lossy ($\sigma > 0$) or is a pure dielectric ($\sigma = 0$), is because most of the available input energy is directed into the Earth by the leaky evanescent surface wave generated by the spherical wave source (the monopole element) in the air medium in proximity to the air-Earth interface [13]. Richmond's method-of-moments model, in solving for the input current $I(z_o)$ indirectly, includes the surface wave and its affect on the far-zone radiation resistance and radiation efficiency. Although this paper is restricted to the calculation of the far-zone field above the Earth, Richmond's moment method analysis for the element and disk current distributions can also be used to calculate the near-zone field including that of the surface wave. This latter effort has not yet been undertaken.

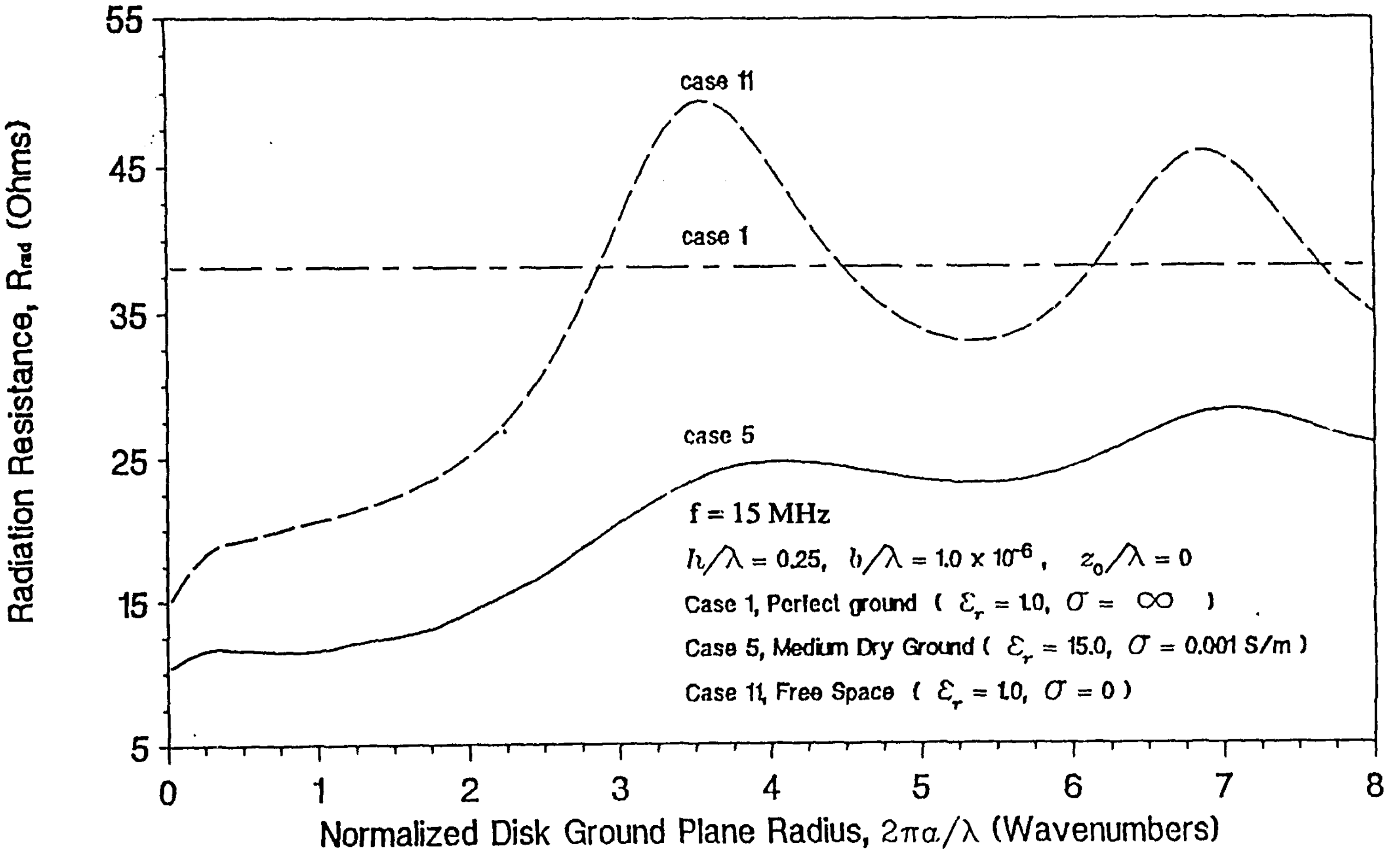


Figure 15. Radiation Resistance

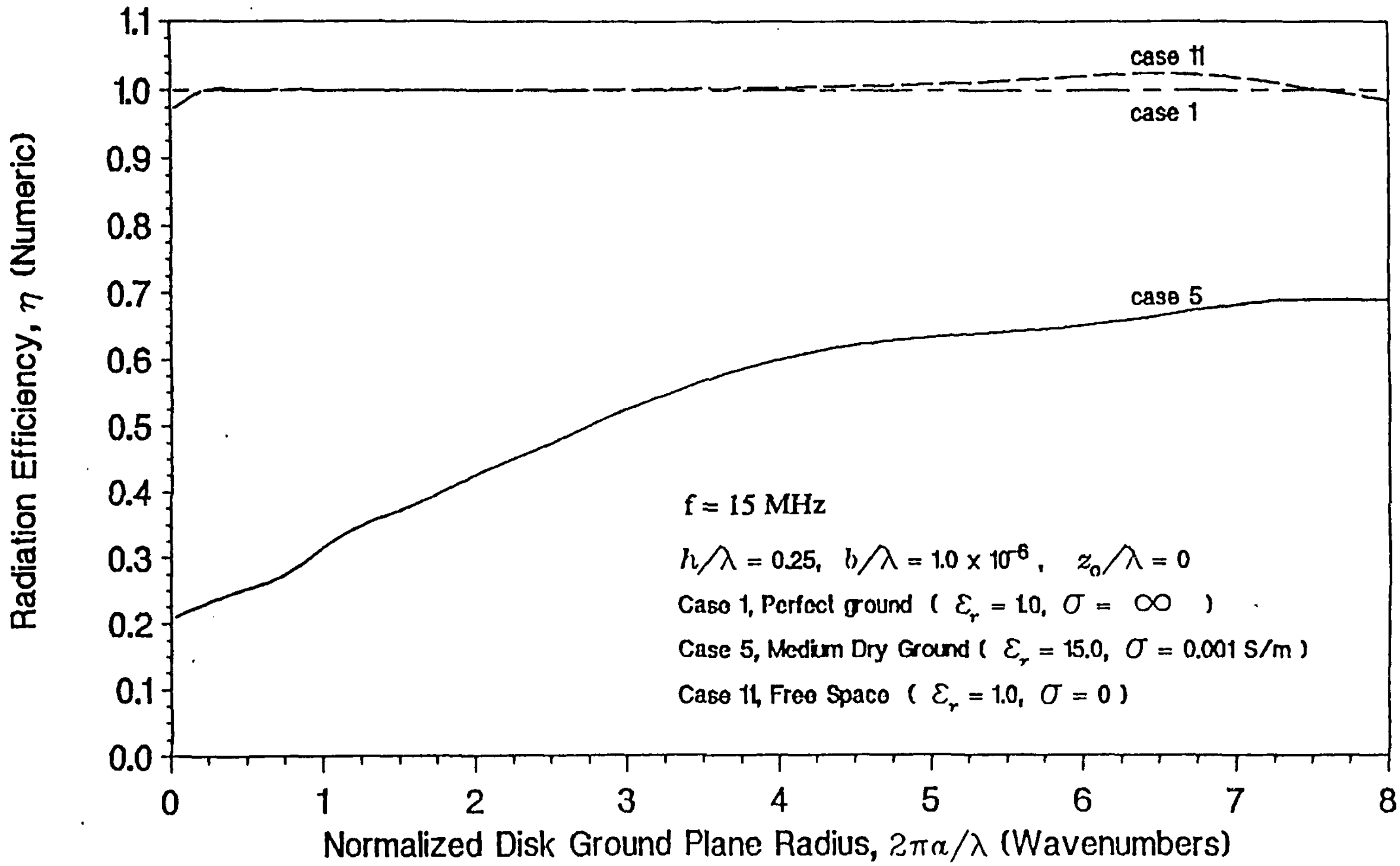


Figure 16. Radiation Efficiency

SECTION 4

VALIDATION OF NUMERICAL RESULTS

Several approaches have been used in validating the numerical results from the programs RICHMOND3 and RICHMOND4. These approaches include validation by comparison with results from the limiting case of disk ground planes in free space; the limiting case of ground planes of zero extent in proximity to Earth; the limiting case of a perfect ground plane of infinite extent; Wait-Surtees model for input impedance; Wait-Walters model for gain; and the Numerical Electromagnetics Code (NEC) for radiation efficiency.

4.1 LIMITING CASE OF DISK GROUND PLANES IN FREE SPACE

In the limiting case of disk ground planes in free space, numerical results from programs RICHMOND3 and RICHMOND4 agree with results from programs RICHMD1 and RICHMD2. The method-of-moments programs RICHMD1 and RICHMD2, for a monopole element on a disk ground plane in free space, have received extensive validation [12]. In reference 12, numerical results for electrically thin monopole elements were compared with results from Brillouin-Stratton induced electromotive force (EMF) method for ground planes of zero extent; Bardeen's integral equation method for ground-plane radii $0 \leq ka \leq 2.75$ wavenumbers; Leitner-Spence method of oblate spheroidal wave functions for ground plane radii $3.0 \leq ka \leq 6.5$ wavenumbers; Awadalla-McClean moment method combined with the geometric theory of diffraction for ground-plane radii $8.5 \leq ka < \infty$ wavenumbers; and the method of images for $ka = \infty$. Consistent and excellent agreements of results were achieved by the RICHMD1 and RICHMD 2 programs.

4.2 LIMITING CASE OF GROUND PLANES OF ZERO EXTENT

In the limiting case of ground planes of zero extent in proximity to Earth, program RICHMOND4 results for the directivity of a quarter-wave monopole element with a disk ground plane of radius $ka = 0.025$ wavenumber resting on medium dry Earth (see Case 5 of figure 2) were compared with results for $ka = 0$ from a Fresnel reflection model (MITRE

Program MODIFIED IMAGES) and Lawrence Livermore Laboratory's method-of-moments program NEC-3 using the Sommerfeld option. Programs RICHMOND4, MODIFIED IMAGES, and NEC-3 gave identical directivity patterns with absolute values of directivity that agreed to within 0.04 dBi. The reason for the close agreement is that the directivity does not depend upon the absolute accuracy of the antenna input current.

Radiation resistance and radiation efficiency do depend upon the absolute accuracy of the antenna input current. RICHMOND4 results of radiation resistance and radiation efficiency, for the above case and various types of Earth, are compared in table 1 with results from NEC-3 (but not MODIFIED IMAGES because the omission of the surface wave in the Fresnel coefficient model affects the radiation efficiency and radiation resistance, but not directivity). The results differ by approximately 10% for radiation resistance and by more than 25% for radiation efficiency. These differences are attributable to the difference in charge density at the base of the monopole element by a factor of 4000 resulting from the different configurations of the two models [16]. In NEC-3, the current produced by the charge distribution is discharged into the Earth through an element of radius 10^{-6} wavelengths, whereas in RICHMOND4 the current is discharged into the Earth through a ground plane of radius 4×10^{-3} wavelengths. The NEC-3 results for the radiation efficiency of a quarter-wave monopole element is augmented by a 128-radial-wire ground plane of radius 0.01 wavelengths (see section 4.6). An increase in the number of monopole segments from 4 to 20 in RICHMOND4 has no significant effect in modifying the table 7 results for radiation efficiency.

4.3 LIMITING CASE OF A GROUND PLANE OF INFINITE EXTENT

In the limiting case of a perfect ground plane of infinite extent, the monopole element of length h may be modeled by the method-of-images as a free-space dipole of half-length h, but with twice the dipole input current, one-half the dipole impedance, twice the dipole directivity in the upper hemisphere, and zero times the dipole directivity in the lower hemisphere. Richmond has written a program, RICHMD6, that uses a sinusoidal-Galerkin method of moments to compute the input impedance, current distribution, and far-zone field

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Table 1. Radiation Resistance and Efficiency of a Vertical, Quarter-Wave, Monopole Element on Flat Earth; $f = 15$ MHz, $b/\lambda = 1.0 \times 10^{-6}$

Earth Classification (ε_r, σ S/m)	Radiation Resistance (Ohms)			Radiation Efficiency (Numeric)		
	*NEC-3	**RICHMD4	***Percent Difference	*NEC-3	**RICHMD4	***Percent Difference
Sea water (70, 5)	34.0	29.5	15.0	0.823	0.799	29.4
Fresh water ($80, 3.0 \times 10^{-2}$)	19.1	17.3	10.4	0.273	0.347	34.3
Wet ground ($30, 1.0 \times 10^{-2}$)	14.5	13.2	10.3	0.144	0.229	36.9
Medium dry ground ($15, 1.0 \times 10^{-3}$)	11.5	10.5	10.3	0.163	0.210	22.2
Very dry ground ($3, 1.0 \times 10^{-4}$)	6.2	5.7	9.6	0.091	0.145	37.6
Pure water, 20°C ($80, 1.7 \times 10^{-3}$)	19.1	17.3	9.4	0.375	0.378	0.8
Ice (−1°C) ($3, 9.0 \times 10^{-5}$)	6.2	5.7	9.6	0.091	0.148	38.8
Ice (−10°C) ($3, 2.7 \times 10^{-5}$)	6.2	5.7	9.5	0.136	0.171	20.8
Average land ($10, 5.0 \times 10^{-3}$)	9.9	9.0	10.3	0.044	0.105	58.3

* Number of element segments, $N = 25$; voltage source excitation at $N = 1$

** Disk ground plane radius, $2\pi a/\lambda = 0.025$ wavenumbers

*** $|(\text{NEC-3} - \text{RICHMD4})/\text{RICHMD4}| \times 100$

of the equivalent free-space dipole. A listing of program RICHMD6 is given in appendix C. Numerical results for input impedance are in reasonable agreement with those from King-Middleton theory [17]. For example, for $h/\lambda = 0.25$ (corresponding to $kh = \pi/2$) and $h/b = 16.56$ (corresponding to $\Omega = 7$), RICHMD6 results for the monopole input impedance are $Z_{in} = 46.52 + j\,15.97$ ohms which differ from the King-Middleton results of $Z_{in} = 47.85 + j\,18.50$ ohms by 2.8 and 13.7% for input resistance and input reactance, respectively. RICHMD6 results for directivity are almost identical to the well-known results for a thin, quarter-wave monopole on a perfect ground plane [12].

4.4 COMPARISON WITH WAIT-SURTEES MODEL FOR INPUT IMPEDANCE

Program RICHMOND4 results for the input impedance of a monopole element with a disk ground plane resting on flat Earth have been compared by Richmond [1] with those obtained from a Wait-Surtees model [18]. In reference 1, the Wait-Surtees results for input reactance are inadvertently given for a disk ground plane in free space rather than for a disk ground plane on flat Earth. RICHMOND4 results for input resistance and input reactance are compared in figures 17 and 18, respectively, with those obtained from a program WAIT-SURTEES written by Richmond and based on the Wait-Surtees model. Program WAIT-SURTEES, described in Appendix D, incorporates results from program RICHMD6 for the input impedance of a monopole element on a perfect ground plane. The RICHMOND4 results are in close agreement with WAIT-SURTEES results, except at small ground-plane radii less than approximately $ka = 1.0$ wavenumber for which the Wait-Surtees model is not accurate. Richmond [10] has compared RICHMD1 results with WAIT-SURTEES results for the input impedance of a monopole element on a disk ground plane in free space and obtained similar agreement as above, but for ground-plane radii greater than approximately $ka = 2.0$ wavenumbers. The RICHMOND4 results in figure 18 for input reactance should not have a local minimum at $ka= 0.75$. A nonconvergent result was obtained at $ka = 0.75$ because of over-segmentation of the disk when the number of disk annular zones was abruptly increased from seven at $ka = 0.5$ to sixteen at $ka = 0.75$.

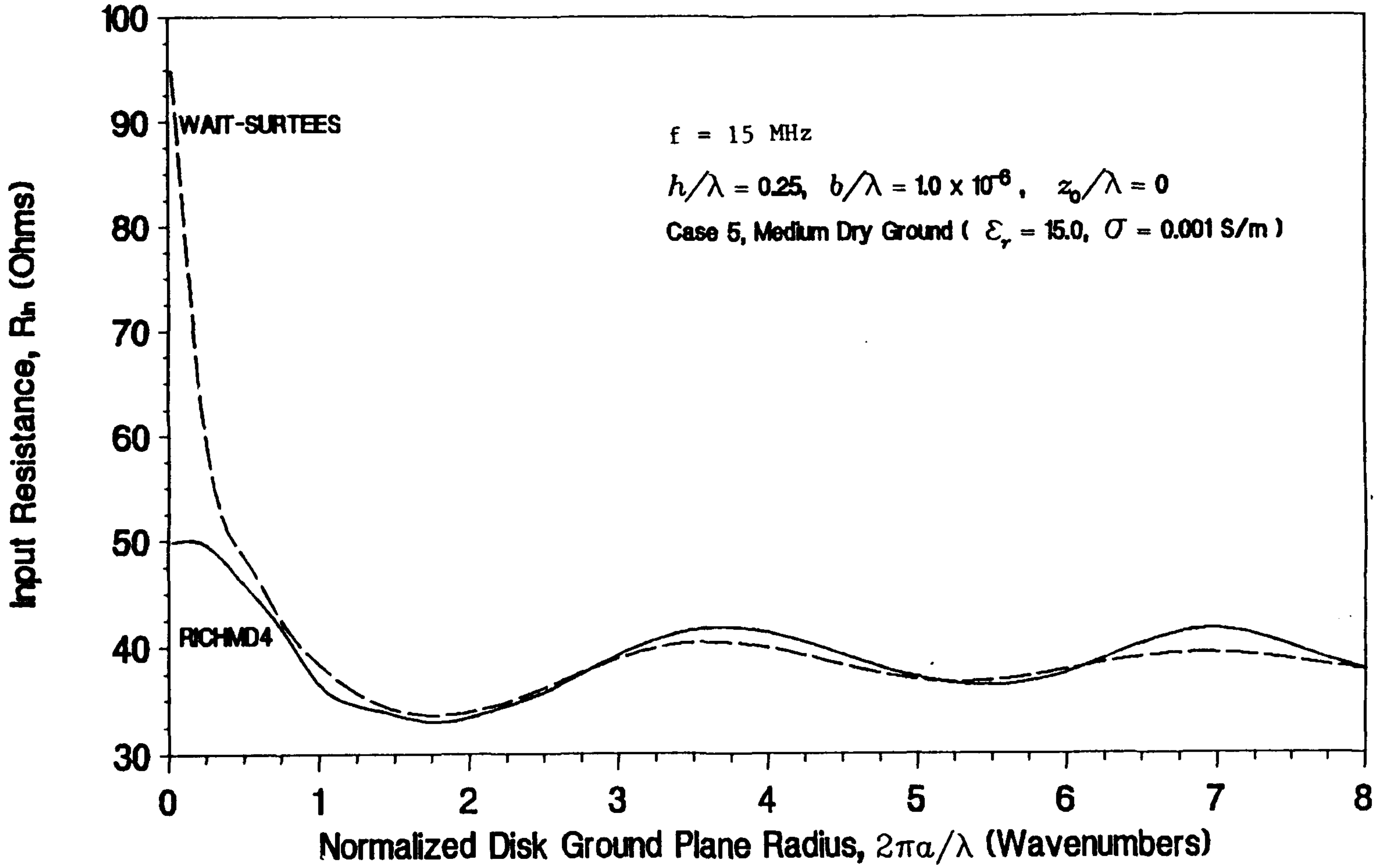

Figure 17. Input Resistance

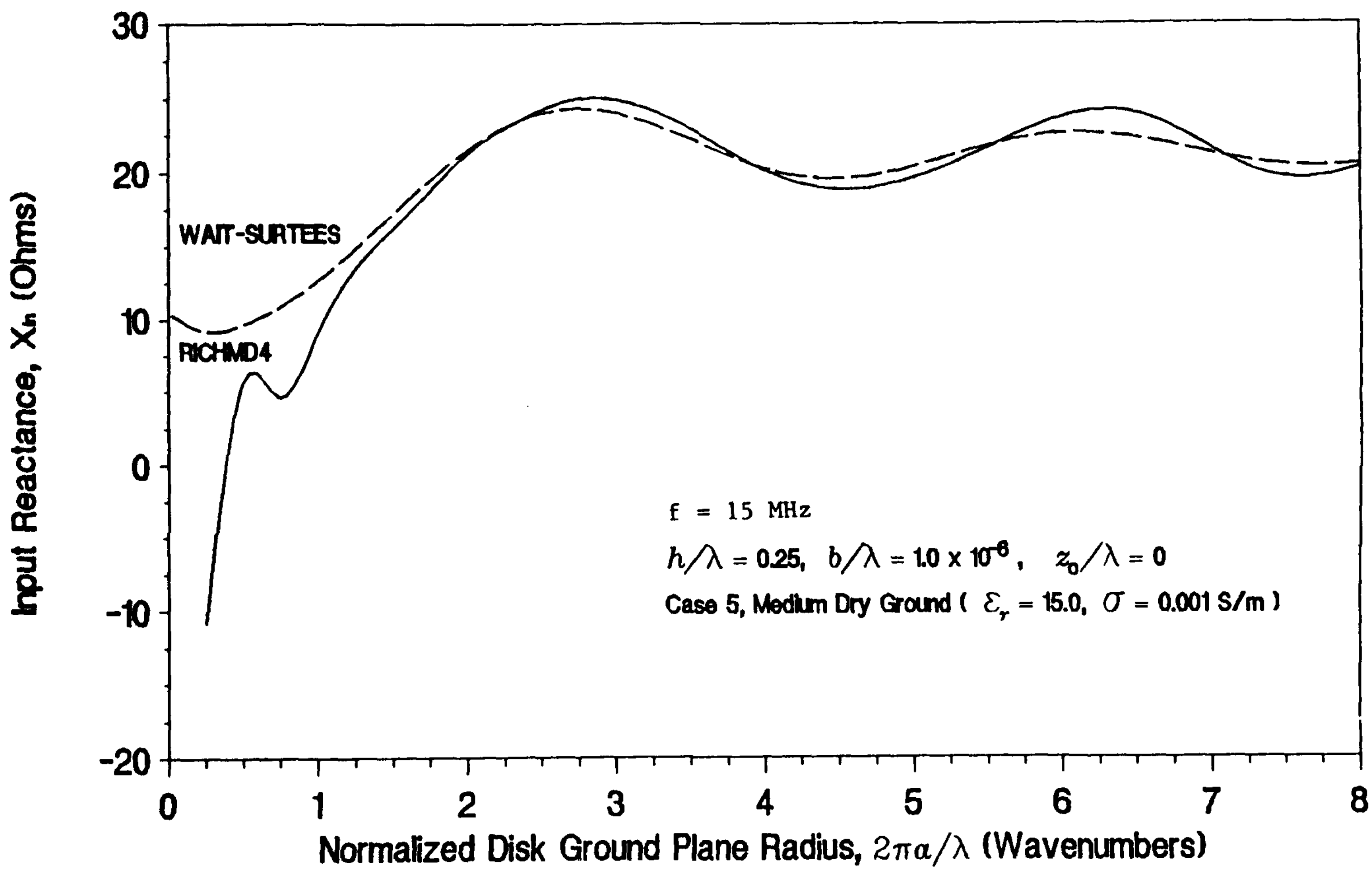

Figure 18. Input Reactance

4.5 COMPARISON WITH WAIT-WALTERS MODEL FOR GAIN

Numerical results of directivity and radiation efficiency from Richmond's method-of-moments program RICHMOND4 cannot be validated against models based on Monteath's compensation theorem [5-8,19] or Sommerfeld's attenuation function [9] because those models yield only the gain (the product of directivity and radiation efficiency) rather than directivity and radiation efficiency as separate entities. Nevertheless, it is of interest to compare RICHMOND4 results for gain with those from the Wait-Walters model [6,7,8,19] based on Monteath's compensation theorem.

First consider the Wait-Walters model. The gain $G(ka, \psi) - G(0, \psi)$ (dB) of an electrically short monopole element on a disk ground plane with radius ka wavenumbers relative to that without a disk ground plane ($ka = 0$) is shown in figure 2 of reference 8 and in figure 23.26 of reference 19 for $ka = 10$, $\varepsilon_r = 9$, and $\sigma = 0$. The Wait-Walters model of reference 8 computes the magnetic field intensity $H(ka, \psi)$ with a disk ground plane as a function of the grazing angle ψ (the complement of the angle of incidence θ) relative to that with no ground plane. At a grazing angle $\psi = 2°$, the Wait-Walters model gives a relative gain of G(10, 2) - G (0, 2) = 4.5 dB

Now consider the Richmond model. Program RICHMOND4 results for a quarter-wave monopole element on a disk ground plane of radius $ka = 8$ wavenumbers on medium dry ground ($\varepsilon_r = 15.0$, $\sigma = 0.001$ *S/m*) gives a directivity at a grazing angle $\psi = 2°$, of $D(8, 2)$ = -8.6 dBi (see figure 12) and a radiation efficiency $\eta = 0.69$ = -1.6 dB (see figure 16). The gain $G(8, 2)$ = -8.6 dBi -1.6 dB = -10.2 dB; for $ka = 0$ and $\psi = 2°$, $D(0,2)$ = -7.9 dBi (see figure 12) and the radiation efficiency $\eta = 0.21$ = -6.8 dB (see figure 16). The gain $G(0,2)$ = -7.9 dBi - 6.8 dBi = -14.7 dBi. The relative gain $G(8,2)$ - G(0,2) = -10.2 dBi + 14.7 dBi = 4.5 dB.

The RICHMOND4 and Wait-Walters results of 4.5 dB for relative gain are identical for these similar cases.

4.6 COMPARISON WITH NEC FOR RADIATION EFFICIENCY

Numerical results of radiation efficiency obtained from programs RICHMOND4, NEC-3, and NEC-GS are compared in figure 19 for the radiation efficiency of a quarter-wave monopole element with small ground planes on or just above medium dry Earth as a function of the ground-plane radius. RICHMOND4 results are for disk ground planes (see figure 16). NEC-3 results are for a ground plane of zero extent (see table 1). NEC-GS results are for radial-wire ground planes whose wires have a radius $b_w = 10^{-5}$ wavelengths [16,20]. The results for disk ground planes are in close agreement with those for ground planes with 128 radial wires.

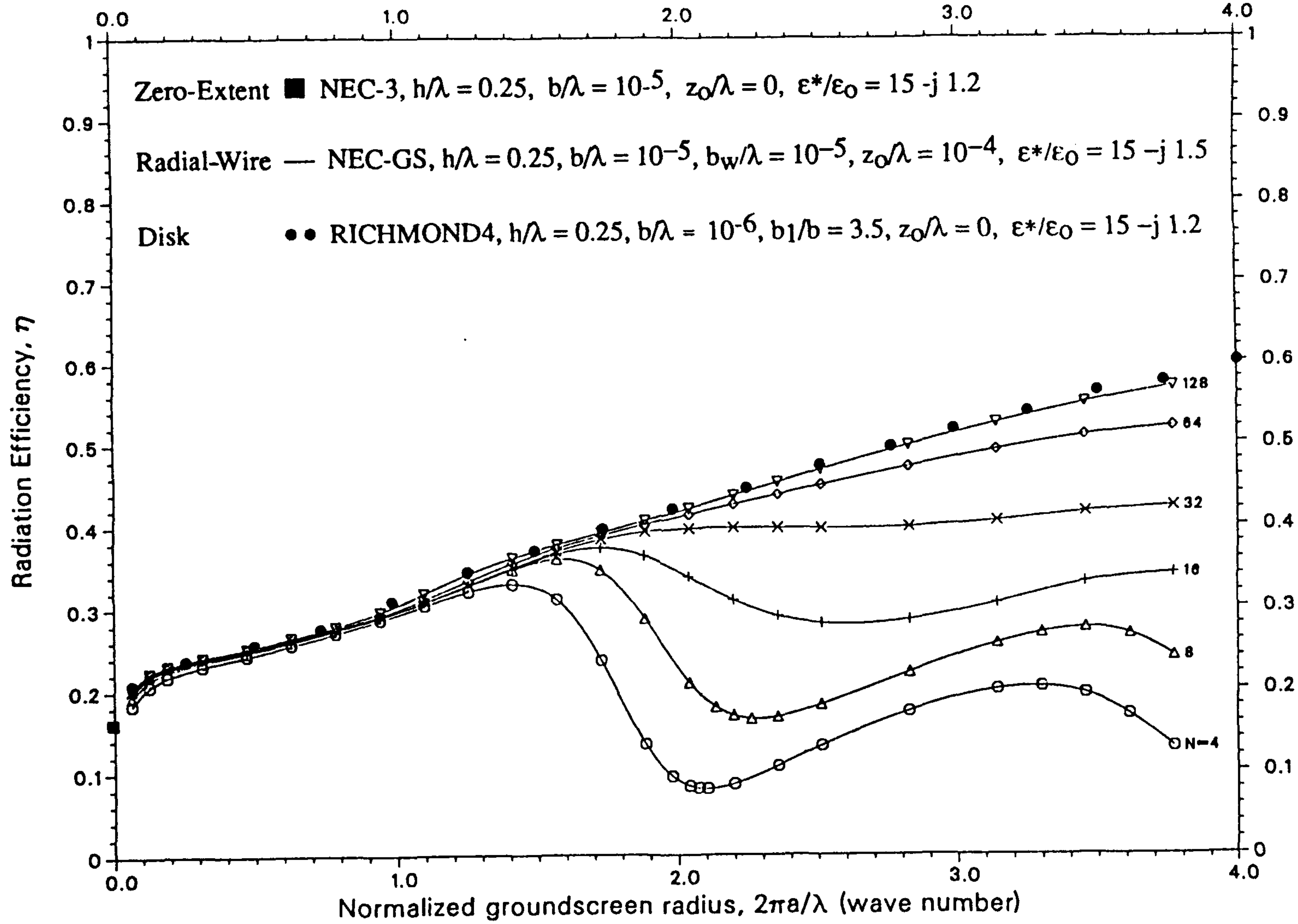

Figure 19. Radiation Efficiency of a Quarter-Wave Monopole Element with Different (Zero-Extent, Radial-Wire, and Disk) Ground Planes on or just above Medium Dry Earth

SECTION 5

CONCLUSIONS

Richmond's moment-method results, for the current distribution and input impedance of a monopole element on a disk ground plane above flat Earth, are used to obtain the far-zone field, directivity pattern, radiation resistance, and radiation efficiency. This model for a disk ground plane complements the NEC method-of-moments model of Burke, et al. for a radial-wire ground plane.

Method-of-moments models, unlike models based on Sommerfeld's attenuation function or variational models based on Monteath's compensation theorem, determine the directivity and radiation efficiency as separate entities rather than lumping them together as a product to yield the antenna gain. Other advantages of the method-of-moments models are more exact determination of current distributions; applicability to electrically small ground planes; direct determination of ground-plane edge diffraction; and avoidance of analytical restrictions on evaluating Sommerfeld's integral. The segmentation of ground planes in method-of-moments models restricts the models to ground planes that are sufficiently small so that computer computational capacity and precision are not exceeded.

The far-zone field in the free-space (air) region is determined as the sum of direct and indirect (reflected from the Earth) fields from the monopole element, disk ground plane, and the magnetic frill of the coaxial-line feed excitation. The far-zone direct fields from the monopole element and disk ground plane are determined from the method-of-moments solution for their current distributions. The far-zone indirect fields are determined using the plane-wave Fresnel reflection coefficient. The significant contribution of the surface wave to the far-zone field at or near the air-Earth interface is not considered, but is small compared to the far-zone field in the direction of peak directivity. However, the significant effect of the surface-wave in determining input current and radiation efficiency are included in the present analysis.

Examples of numerical results are presented for the directivity pattern, peak directivity, radiation resistance, and radiation efficiency of a thin, quarter-wave monopole element on small to moderately large disk ground planes (of radius 0 to 8 wavenumbers) resting on medium dry Earth. Results are compared with these for a ground plane of infinite extent and for ground planes in free space. In the presence of Earth, the directivity patterns are approximately independent of disk radius for ground-plane radii at least as large as eight wavenumbers. The peak directivity is within 0.5 dBi of that for a perfect ground plane. The direction of peak directivity is approximately 30° above the horizon. The directivity at angles of incidence of 82°, 84°, 86°, 88°, and 90° are approximately 4 dB, 5 dB, 7 dB, 13 dB, and ∞ dB, respectively, below the peak directivity. The numeric directivity is given approximately by the empirical expression $10 \cos \theta \sin^3 \theta$ in the hemisphere above the Earth and by zero in the hemisphere below the Earth. The radiation efficiency increases monotonically with increasing disk radius in the presence of Earth: from 0.21 for a ground plane of zero extent to 0.69 for a ground-plane radius of eight wavenumbers.

Numerical results from Richmond's method-of-moments computer programs RICHMOND3 and RICHMOND4 for a monopole element with a disk ground plane above flat Earth are in good agreement with results known from other models in the limiting cases of disk ground planes in free space, disk ground planes of zero extent in proximity to Earth, and a perfect ground plane. RICHMOND3 results for input impedance are in good agreement with results from a Wait-Surtees variational model, except for ground-plane radii less than approximately one wavenumber for which the Wait-Surtees model is not accurate. A RICHMOND4 result for antenna gain is in agreement with a result from a Wait-Walters variational model. RICHMOND4 results of radiation efficiency are in close agreement with NEC-GS method-of-moment results for ground planes with a large number of radial wires.

LIST OF REFERENCES

1. Richmond, J. H., June 1985, "Monopole Antenna on Circular Disk Over Flat Earth," *IEEE Trans. Antennas and Propagation,* AP-33, No. 6, pp. 633-637.

2. Burke, G. J., and A. J. Poggio, January 1981, "Numerical Electromagnetics Code (NEC) -- Method of Moments," Lawrence Livermore National Laboratory, Report UCID 18834.

3. Burke, G. J. and E. K. Miller, 1984, "Modeling Antennas Near to and Penetrating a Lossy Interface," *IEEE Trans. Antennas and Propagation,* Volume AP-32, pp. 1040-1049.

4. Burke, G. J., October 1983, *User's Guide Supplement for NEC-3 for Modeling Buried Wires*, Lawrence Livermore National Laboratory, Report UCID - 19918.

5. Wait, J. R. and W. A. Pope, 1954, "The Characterization of a Vertical Antenna with a Radial Conductor Ground System," *Applied Scientific Research,* Volume 4, Section B, pp. 177-195 (The Hague).

6. Wait, J. R., and and L. C. Walters, April 1963, "Influence of a Sector Ground Screen on the Field of a Vertical Antenna," *U. S. National Bureau of Standards,* Monograph 60, pp. 1-11.

7. Hill, D. A. and J. R. Wait, January 1973, "Calculated Pattern of a Vertical Antenna with a Finite Radial-Wire Ground System," *Radio Science,* Volume 8, No. 1, pp. 81-86.

8. Wait, J. R., 15-17 August 1990, "Low Angle Radiation from HF Antennas," *Symposium on Antenna Technology and Applied Electromagnetics*, Winnipeg, Canada.

9. Rafuze, R. P. and J. Ruze, December 1975, "Low Angle Radiation from Vertically Polarized Antennas Over Radially Heterogeneous Flat Ground," *Radio Science,* Volume 10, pp. 1011-1018.

10. Richmond, J . H., December 1984, "Monopole Antenna on Circular Disk," *IEEE Trans. Antennas and Propagation,* AP-35, No. 5, pp. 488-495.

11. Weiner, M. M., May 1987, "Monopole Element at the Center of a Circular Ground Plane whose Radius is Small or Comparable to a Wavelength," *IEEE Trans. Antennas and Propagation,* AP-35, No. 5, pp. 488-495.

12. Weiner, M. M., and S. P., Cruze, C. C. Li, and W. J. Wilson, 1987, *Monopole Elements on Circular Ground Planes,* Norwood, MA: Artech House.

LIST OF REFERENCES (CONCLUDED)

13. Weiner, M. M., September 1991, *Performance of Ground-Based High-Frequency Receiving Arrays with Electrically-Small Ground Planes*, MTR-11277, The MITRE Corporation, Bedford, MA. Also available from National Technical Information Service as ADA250489.

14. Tsai, L. L., 1970, *Analysis and Measurement of a Dipole Antenna Mounted Symmetrically on a Conducting Sphere or Cylinder*, Ph.D. dissertation, Department of Electrical Engineering, Ohio State University, Columbus, Ohio.

15. Tsai, L. L., September 1972, "A Numerical Solution for the Near and Far Fields of an Annular Ring of Magnetic Current," *IEEE Trans. Antennas and Propagation*, Vol. AP-20, No. 5, pp. 569-576.

16. Weiner, M. M., September 1991, *Validation of the Numerical Electromagnetics Code (NEC) for Antenna Wire Elements in Proximity to Earth*, MTR-11278, The MITRE Corporation, Bedford, MA. Also available from National Technical Information Service as AD A250507.

17. King, R. W. P., 1956, *The Theory of Linear Antennas*, Cambridge, MA, Harvard University Press, pp. 158-159.

18. Wait, J. R. and W. J. Surtees, May 1954, "Impedance of a top-loaded antenna of arbitrary length over a circular ground screen," *Journal of Applied Physics*, Volume 25, pp. 553-555.

19. Wait, J. R., 1969, "Characteristics of Antennas Over Lossy Earth," Chapter 23, *Antenna Theory*, Part 2, New York: McGraw-Hill, R. E. Collin and F. J. Zucker, Editors, pp. 424-437.

20. Weiner, M. M. and Zamoscianyk, S., September 1991, *Radiation Efficiency and Input Impedance of Monopole Elements with Radial-Wire Ground Planes in Proximity to Earth*, M91-104, The MITRE Corporation, Bedford, MA. Also available from National Technical Information Service as AD A244578.

21. Weiner, M. M., June 1992, *Monopole Elements with Disk Ground Planes on Flat Earth: Atlas of Directivity, Radiation Efficiency, Radiation Resistance, and Input Impedance*, MTR-92B0000089, The MITRE Corporation, Bedford, MA.

22. Carson, J. R., July 1924, "A Generalization of the Reciprocal Theorem," *Bell Systems Tech. J.*, Vol. 3, pp. 393-399.

23. Carson, J. R., June 1929, "Reciprocal Theorems in Radio Communication," *Proc. I.R.E.*, Vol. 17, pp. 952-956.

APPENDIX A

COMPUTER PROGRAM RICHMOND3 FOR THE INPUT IMPEDANCE AND CURRENT DISTRIBUTION OF A MONOPOLE ELEMENT ON A DISK GROUND PLANE ABOVE FLAT EARTH

COMPUTER PROGRAM RICHMOND3
MONOPOLE ANTENNA ON CIRCULAR DISK OVER FLAT EARTH

(Current Distribution and Impedance)

by

Jack H. Richmond

January 29, 1990

INTRODUCTION[1]

Appendix I presents the computer program RICHMOND3 together with all the necessary subroutines. This FORTRAN program calculates the current distribution and impedance of a monopole antenna mounted at the center of a circular disk over the flat lossy earth.

See: [J. H. Richmond, "Monopole Antenna on Circular Disk Over Flat Earth," IEEE Transactions, Vol. AP-33, pp. 633-637, June 1985.]

Comment statements have been inserted in the main program and the subroutines to assist the user. Only a few brief comments will be required in this Introduction.

RICHMOND3 performs all calculations with double precision. The theoretical basis for this program is presented in the above published paper, and the notation in the program corresponds with the notation in the paper with one exception. The outer radius of the disk is denoted by b in the program and by c in the paper.

In Appendix I, Table I presents the antenna impedance as calculated with RICHMOND3 on a VAX computer for the following disk radii: BL = 0.1, 0.2, 0.3 and 0.4. The antenna impedance (in free space and on a lossy flat earth) agree closely with the original calculations obtained on a DATACRAFT computer in May 1979. Table II presents the current distributions on the monopole and the disk (in free space and on a lossy

[1] Appreciation is expressed to The MITRE Corporation for sponsoring this report. The computer program RICHMOND3 was developed (in single precision) in 1979 with other sponsorship.

flat earth) with BL = 0.1. These results also agree closely with the original calculations of May 1979.

Table III presents the antenna impedance (in free space and on a flat earth) as calculated with a VAX with single precision. Comparison with Table I indicates that the need for double precision is marginal for this case.

In the original program, subroutine CROUT was employed to solve the simultaneous linear equations. In RICHMOND3, CROUT is replaced with CMINV which employs full pivoting (on rows and columns) whereas CROUT does not pivot. On the other hand CMINV is presumably slower (in solving large matrix equations) because it inverts the matrix, whereas CROUT solves the equations without inverting.

The current distribution will be printed from CMINV if IWCJ = 1, but the printout will be suppressed if IWCJ = 0.

Diagnostic data will be printed from several subroutines if IWZ = 1. This printout is suppressed if IWZ = 0.

The integer NPH controls the numerical integrations in several subroutines. NPH determines the number of times the integrand is to be sampled with Simpson's rule. The value NPH = 6 usually gives a suitable compromise between accuracy and computational expense. A larger value will increase the expense, and it may improve the accuracy in some cases.

TL denotes the thickness of the circular disk, measured in free-space wavelengths. To promote convergence of the moment method (as NEQ is increased), the value TL = AL/100 is recommended regardless of the true thickness of the metallic circular disk. This result is rather unexpected, and the interpretation is not totally understood. Of course, it is assumed that the disk thickness TL (as well as the monopole wire radius AL) is much smaller than the wavelength.

Appendix I. RICHMOND3 Program Listing

```
C                                      1
C     RICHMOND3
C     MONOPOLE AT CENTER OF CIRCULAR DISK OVER FLAT EARTH.     RICHMOND3.1
C     DOUBLE PRECISION.
C     CURRENT DISTRIBUTION AND IMPEDANCE.
C     SEE:  RICHMOND, "MONOPOLE ANTENNA ON CIRCULAR DISK OVER FLAT EARTH",
C     IEEE TRANS., VOL. AP-33, PP. 633-637, JUNE 1985.
C     LINK:    BES10,CISI,CMINV,DZ11,DZDD,DZWD,DZWW,EXPJ,GRILL,
C              QDD,QDM,QMM,SKEW,SKEWS,SKEWT,SPART,ZSDM,ZSMM
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 CJ(30),VJ(30),ZJ(30),VIJ(30,30),ZIJ(30,30)
      COMPLEX*16 Y11,DET,EC,D11,D12,D21,D22,DZ1J,DV1,W11
      COMPLEX*16 P11,P12,P21,P22,ZDD,ZDM,ZMD,ZMM,Z11,ZD,Z22,Z12,Z21
      DIMENSION FB(500),LLL(30),MMM(30)
      DATA E0,U0/8.85418533677E-12,1.25663706144E-6/
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      DATA ICC,IFB/30,500/
    1 FORMAT(1X,2I5,7E15.4)
    2 FORMAT(1X,7F17.8)
    5 FORMAT(1H0)
C     AL = RADIUS OF WIRE IN WAVELENGTHS.
C     HL = LENGTH OF MONOPOLE IN WAVELENGTHS.
C     FMC = FREQUENCY IN MEGAHERTZ.
C     BAR = RADIUS RATIO FOR COAXIAL FEED CABLE.
C     BL = RADIUS OF CIRCULAR DISK IN WAVELENGTHS = EPSLN/TP.
C     NSD = NUMBER OF SEGMENTS ON THE DISK.
C     NSW = NUMBER OF SEGMENTS ON THE WIRE.
C     HDL = HEIGHT OF DISK ABOVE THE EARTH IN WAVELENGTHS.
C     ER = RELATIVE PERMITTIVITY OF EARTH.
C     SIG = CONDUCTIVITY OF EARTH, MHO/M.
C     SET IWCJ=1 TO WRITE THE CURRENT DISTRIBUTION CJ(N),
C      OR IWCJ=0 TO SUPPRESS WRITEOUT OF CJ(N).
C     SET HDL = NEGATIVE, FOR MONOPOLE-DISK IN FREE SPACE,
C      OR HDL = POS. FOR FREE SPACE + FLAT EARTH.
C     SET DTHD = NEGATIVE, TO SKIP THE GAIN CALCULATIONS.
C
C     TL = 1.D-5 FOR EPSLN GREATER THAN OR EQUAL 0.25,
C        = AL.D-4 FOR EPSLN LESS THAN 0.25.
      AL=.003
      BAR=3.
      BL=.1
      ER=4.
      FMC=300.
      HDL=.0
      HL=.229
      SIG=.001
      TL=AL/100.
      IWCJ=1
      [illegible]
      IWZ=0
      NPH=6
      NSD=10
      NSW=4
      NEQ=NSD+NSW-1
      AK=TP*AL
      BK=TP*BL
      HK=TP*HL
      HDK=TP*HDL
      TK=TP*TL
      OMEG=TP*FMC*1.E6
      EC=DCMPLX(ER,-SIG/(OMEG*E0))
      DKD=(BK-AK)/NSD
      DKW=HK/NSW
      RH2=AK+DKD
      IF(RH2.LT.BAR*AK)GO TO 400
      TDKD=2.*DKD
      CDKD=DCOS(DKD)
      SDKD=DSIN(DKD)
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      MAX=NSW-1
      MA=NSD+1
      CALL QMM(AK,DKD,DKW,CDKD,SDKD,CDK,SDK,TK,IWZ,NPH,Z11)
      ZIJ(1,1)=Z11
```

```
      IF(NSD.LE.1)GO TO 100
      S1=AK
      DO 60 J=2,NSD
      S2=S1+DKD
      S3=S1+TDKD
      T1=AK
      DO 50 I=2,J
      T2=T1+DKD
      T3=T1+TDKD
      CALL QDD(CDKD,SDKD,S1,S3,T1,T3,TK,IWZ,NPH,Z22)
      ZIJ(I,J)=Z22
   50 T1=T1+DKD
      CALL QDM(AK,DKD,DKW,CDKD,SDKD,SDK,S1,S3,TK,IWZ,NPH,Z12)
      ZIJ(1,J)=Z12
   60 S1=S1+DKD
  100 IF(NSW.LE.1)GO TO 200
      CALL SPART(AK,DKD,DKW,MAX,IWZ,ZJ,CJ)
      L=0
      DO 160 I=NA,NEQ
      DO 150 J=I,NEQ
      K=J-I+1
  150 ZIJ(I,J)=ZJ(K)
      L=L+1
      ZIJ(1,I)=CJ(L)
  160 CONTINUE
  178 IF(NSD.LE.1)GO TO 200
      Z2=.0
      DO 190 J=NA,NEQ
      Z2=Z2+DKW
      S1=Z2-DKW
      S3=Z2+DKW
      RH2=AK
      DO 180 I=2,NSD
      RH2=RH2+DKD
      T1=RH2-DKD
      T3=RH2+DKD
      CALL SKEWT(AK,S1,S3,T1,T3,CDK,SDK,CDKD,SDKD,IWZ,Z12)
  180 ZIJ(I,J)=Z12
  190 CONTINUE
  200 CALL GRILL(AK,BAR,DKD,DKW,NEQ,NSD,NSW,TK,VJ)
      DO 210 I=1,NEQ
      DO 210 J=I,NEQ
      WRITE(17,1)I,J,ZIJ(I,J)
      ZIJ(J,I)=ZIJ(I,J)
  210 VIJ(I,J)=ZIJ(I,J)
      WRITE(17,5)
      CALL CMINV(CJ,VJ,ZIJ,ICC,IWCJ,1,LLL,MMM,NEQ,DET)
      Y11=CJ(1)
      Z11=1./Y11
      WRITE(6,2)AL,BL,HL,Z11
      WRITE(17,2)AL,BL,HL,Z11
      WRITE(17,5)
C     FOR MONOPOLE ON DISK IN FREE SPACE, SKIP TO STATEMENT 320.
C
      CALL DZ11(AK,BAR,DKD,DKW,EC,FB,HDK,TK,IFB,D11,DV1)
      ZIJ(1,1)=D11
      IF(NSD.LE.1)GO TO 265
      S2=AK+DKD
      DO 260 J=2,NSD
      T2=AK+DKD
      I12=1
      DO 250 I=2,J
      CALL DZDD(AK,DBET,DKD,DKW,EC,FB,HDK,S2,T2,TK
     2,IFB,I12,KMX,D12,D22)
      IF(I.EQ.2)P12=D12
      ZIJ(I,J)=D22
```

```
      I12=2
  250 T2=T2+DKD
      ZIJ(1,J)=P12
  260 S2=S2+DKD
  265 IF(NSW.LE.1)GO TO 278
      DO 276 K=1,MAX
      CALL DZWW(AK,DKD,DKW,EC,HDK,K,TK,ZJ,DZ1J)
      J=NA+K-1
      ZIJ(1,J)=DZ1J
      L=1
      DO 270 I=NA,J
      ZIJ(I,J)=ZJ(L)
  270 L=L+1
  276 CONTINUE
  278 IF(NSD.LE.1)GO TO 300
      Z2=.0
      DO 290 J=NA,NEQ
      Z2=Z2+DKW
      CALL DZWD(AK,DKD,DKW,EC,HDK,NSD,TK,Z2,ZJ)
      DO 280 I=2,NSD
  280 ZIJ(I,J)=ZJ(I)
  290 CONTINUE
  300 DO 310 I=1,NEQ
      DO 308 J=I,NEQ
      Z12=VIJ(I,J)
      D12=ZIJ(I,J)
      WRITE(17,1)I,J,Z12,D12
      ZIJ(I,J)=Z12+D12
  308 CONTINUE
  310 CONTINUE
      WRITE(17,5)
      VJ(1)=VJ(1)+DV1
      DO 315 I=1,NEQ
      DO 312 J=I,NEQ
  312 ZIJ(J,I)=ZIJ(I,J)
  315 CONTINUE
      CALL CMINV(CJ,VJ,ZIJ,ICC,IWCJ,1,LLL,MMM,NEQ,DET)
      Y11=CJ(1)
      W11=1./Y11
      WRITE(6,2)Z11,W11
      WRITE(17,1)NSD,NSW,AL,BL,HDL
      WRITE(17,5)
      WRITE(17,2)Z11,W11
  320 CONTINUE
  400 CONTINUE
  500 CALL EXIT
      END
C
C
```

TABLE I
DOUBLE PRECISION

NSD	NSW	AL	BAR	ER	FMC	HDL	HL	SIG
10	4	0.003	3.	4.	300	.0	0.229	.001

BL	ANTENNA IMPEDANCE Z11 (in free space)		ANTENNA IMPEDANCE Z11 (on flat earth)	
	R11	X11	R11	X11
0.1	14.8427	-53.8714	29.5665	-32.5406
0.2	17.4560	-21.1708	25.4167	-19.7828
0.3	20.5451	-9.0883	23.5883	-9.5982
0.4	26.8578	-1.0801	29.5489	-4.6751

5

Current Distribution on monopole on Circular Disk in Free Space.

TABLE II
DOUBLE PRECISION

I	CJ(I) (norm)	CJ(I) (mag.)	CJ(I) (phase)
1	1.000	0.0178959	74.6
2	0.983	0.0175990	-105.5
3	0.946	0.0169306	-105.9
4	0.908	0.0162422	-106.1
5	0.857	0.0153451	-106.3
6	0.800	0.0143135	-106.4
7	0.729	0.0130419	-106.6
8	0.642	0.0114806	-106.7
9	0.528	0.0094462	-106.8
10	0.395	0.0070747	-106.9
11	0.843	0.0150935	72.6
12	0.648	0.0116011	71.4
13	0.386	0.0069004	70.4

Current Distribution on Monopole on Circular Disk on Flat Earth.

I	CJ(I) (norm)	CJ(I) (mag.)	CJ(I) (phase)
1	1.000	0.0227445	47.7
2	0.987	0.0224556	-132.2
3	0.957	0.0217627	-132.5
4	0.909	0.0206744	-132.6
5	0.845	0.0192282	-132.7
6	0.782	0.0177918	-132.6
7	0.713	0.0162147	-132.1
8	0.616	0.0140152	-131.3
9	0.470	0.0107005	-130.3
10	0.293	0.0066696	-129.4
11	0.880	0.0200169	43.9
12	0.686	0.0155928	42.0
13	0.410	0.0093323	40.4

NSD	NSW	AL	BL	HDL
10	4	0.3000E-02	0.1000E+00	0.0000E+00

Antenna Impedance Z11 (in free space) R	X	Antenna Impedance Z11 (on flat earth) R	X
14.84274593	-53.87146810	29.56653347	-32.54060135

5a

TABLE III

SINGLE PRECISION

NSD	NSW	AL	BAR	ER	FMC	HDL	HL	SIG
10	4	.003	3.	4.	300.	.0	0.229	.001

BL	ANTENNA IMPEDANCE Z11 (in free space)		ANTENNA IMPEDANCE Z11 (on flat earth)	
	R11	X11	R11	X11
0.1	14.8184	-53.8642	29.5429	-32.5342
0.2	17.4301	-21.1669	25.3907	-19.7795
0.3	20.5567	-9.0920	23.5997	-9.6021
0.4	26.8575	-1.0805	29.5487	-4.6758

```
C                                    6
C
      SUBROUTINE BES10(XX,B,B1,ID)                                 BES10
C     B  = BESSEL FUNCTION J sub 0 with real argument XX.
C     B1 = BESSEL FUNCTION J sub 1 with real argument XX.
C     SET ID = (0, 1, 2) TO CALCULATE (J sub o, J sub 1, or both).
      IMPLICIT REAL*8 (A-H), (P-Z)
      B=1.
      B1=.0
      IF(XX.EQ..0)RETURN
      X=DABS(XX)
      IF(X.GT..01)GO TO 10
      X2=X*X
      X4=X2*X2
      B =1.-X2/4.+X4/64.
      B1=X*(1.-X2/8.)/2.
      RETURN
   10 DX=X
      IF(X.GE.3.)GO TO 100
      C=DX*DX/9.
      IF(ID.EQ.1)GO TO 20
      B  =(((((.21D-3*C-.39444D-2)*C+.444479D-1)*C-.3163866)*C+.1265
     16208D+1)*C-2.2499997)*C+1.
      IF(ID.EQ.0)RETURN
   20 B1 =((((((.1109D-4*C-.31761D-3)*C+.443319D-2)*C-.3954289D-1)*C+
     1.21093573)*C-.56249985)*C+.5)*DX
      RETURN
  100 D=3./DX
      C=1./DSQRT(DX)
      IF(ID.EQ.1)GO TO 120
      EA=C*((((((.14476D-3*D-.72805D-3)*D+.137237D-2)*D-.9512D-4)*D-
     4.552740D-2)*D-.77D-6)*D+.797884560803)
      FA=(((((.13558D-3*D-.29333D-3)*D-.54125D-3)*D+.262573D-2)*D-
     5.3954D-4)*D-.4166397D-1)*D-.785398163397+DX
      B  =EA*DCOS(FA)
      IF(ID.EQ.0)RETURN
  120 EB=C*((((((-.20033D-3*D+.113653D-2)*D-.249511D-2)*D+.17105D-3)*D+
     6.1659667D-1)*D+.156D-5)*D+.797884560803)
      FB=(((((-.29166D-3*D+.79824D-3)*D+.74348D-3)*D-.637879D-2)*D+
     7.5650D-4)*D+.12499612)*D-2.356194490192+DX
      B1 =EB*DCOS(FB)
      RETURN
      END
C
C
```

```
C                                  7
C
      SUBROUTINE CISI(CI,CIN,SI,X)                              CISI
C     CALCULATES CI = COSINE INTEGRAL, AND
C     SI = SINE INTEGRAL WITH ARGUMENT X.
      IMPLICIT REAL*8 (A-H), (P-Z)
      DATA GAM,P2/.57721566,1.57079632/
      A=DABS(X)
      IF(A.GT.4.)GO TO 10
      IF(A.GT..1)GO TO 3
      IF(A.GT.0.)GO TO 2
      CI=.0
      CIN=.0
      SI=.0
      RETURN
    2 X2=A*A
      SI=X*((.03*X2-1.)*X2/18.+1.)
      CIN=.25*X2*((X2/45.-1.)*X2/24.+1.)
      GO TO 8
    3 Y=(4.-A)*(4.+A)
      SI=X*(((((1.753141D-9*Y+1.568988D-7)*Y+1.374168D-5)*Y+6.939889D-4)
     C*Y+1.964882D-2)*Y+4.395509D-1)
      CIN=      A*A*(((((1.386985D-10*Y+1.584996D-8)*Y
     C+1.725752D-6)*Y+1.185999D-4)*Y+4.990920D-3)*Y+1.315308D-1)
    8 CI=GAM+DLOG(A)-CIN
      RETURN
   10 SI=DSIN(A)
      Y=DCOS(A)
      Z=4./A
      U=((((((((4.048069D-3*Z-2.279143D-2)*Z+5.515070D-2)*Z-7.261642D-2)
     C*Z+4.987716D-2)*Z-3.332519D-3)*Z-2.314617D-2)*Z-1.134958D-5)*Z
     C+6.250011D-2)*Z+2.583989D-10
      V=((((((((-5.108699D-3*Z+2.819179D-2)*Z-6.537283D-2)*Z
     C+7.902034D-2)*Z-4.400416D-2)*Z-7.945556D-3)*Z+2.601293D-2)*Z
     C-3.764000D-4)*Z-3.122418D-2)*Z-6.646441D-7)*Z+2.5D-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)+P2
      IF(X.LT..0)SI=-SI
      CIN=GAM+DLOG(A)-CI
      RETURN
      END
C
C
```

```
C                                  8
      SUBROUTINE CMINV(C,V,Z,IDM,IWR,I12,L,M,NEQ,DET)                CMINV.1
C     CMINV2 INVERTS THE MATRIX Z(I,J) AND SOLVES THE
C     SIMULTANEOUS LINEAR EQUATIONS TO DETERMINE C(I).
C     V(I) = EXCITATION COLUMN.
C     Z(I,J) = IMPEDANCE MATRIX.
C     IDM = DIMENSION OF Z(IDM,IDM) IN CALLING PROGRAM.
C     IWR = 1 IF SOLUTION IS TO BE PRINTED.
C     IWR = 0 IF PRINTOUT IS TO BE SUPPRESSED.
C     I12 = 1 ON FIRST CALL, WHERE CMINV MUST INVERT Z.
C     I12 = 2 ON LATER CALLS, IF Z(I,J) HAS ALREADY BEEN INVERTED.
C     L(I), M(I) = WORK ARRAYS.
C     NEQ = NUMBER OF SIMULTANEOUS LINEAR EQUATIONS.
C     DET = DETERMINANT OF THE SQUARE MATRIX.
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 C(1),V(1),S
      COMPLEX*16 Z(IDM,IDM),BIGZ,HOLD,DET
      DIMENSION L(1),M(1)
    2 FORMAT(1X,I5,F10.3,F15.7,F10.1)
    5 FORMAT(1H0)
      N=NEQ
      IF(I12.NE.1)GO TO 150
C     DET=DCMPLX(1.D0,0.D0)
      DO 80 K=1,N
      L(K)=K
      M(K)=K
      BIGZ=Z(K,K)
      DO 20 J=K,N
      DO 20 I=K,N
   10 IF(CDABS(BIGZ)-CDABS(Z(I,J)))15,19,19
   15 BIGZ=Z(I,J)
      L(K)=I
      M(K)=J
   19 CONTINUE
   20 CONTINUE
      J=L(K)
      IF(J-K)35,35,25
   25 CONTINUE
      DO 30 I=1,N
      HOLD=-Z(K,I)
      Z(K,I)=Z(J,I)
   30 Z(J,I)=HOLD
   35 I=M(K)
      IF(I-K) 45,45,38
   38 CONTINUE
      DO 40 J=1,N
      HOLD=-Z(J,K)
      Z(J,K)=Z(J,I)
   40 Z(J,I)=HOLD
   45 CONTINUE
      DO 55 I=1,N
      IF(I-K)50,55,50
   50 Z(I,K)=Z(I,K)/(-BIGZ)
   55 CONTINUE
      DO 65 I=1,N
      DO 65 J=1,N
      IF(I-K)60,64,60
   60 IF(J-K)62,64,62
   62 Z(I,J)=Z(I,K)*Z(K,J)+Z(I,J)
   64 CONTINUE
   65 CONTINUE
      DO 75 J=1,N
      IF(J-K)70,75,70
   70 Z(K,J)=Z(K,J)/BIGZ
   75 CONTINUE
C     DET=DET*BIGZ
```

```
      Z(K,K)=1./BIGZ
   80 CONTINUE
      K=N
  100 K=K-1
      IF(K)150,150,105
  105 I=L(K)
      IF(I-K)120,120,108
  108 CONTINUE
      DO 110 J=1,N
      HOLD=Z(J,K)
      Z(J,K)=-Z(J,I)
  110 Z(J,I)=HOLD
  120 J=M(K)
      IF(J-K)100,100,125
  125 CONTINUE
      DO 130 I=1,N
      HOLD=Z(K,I)
      Z(K,I)=-Z(J,I)
  130 Z(J,I)=HOLD
      GO TO 100
  150 CMX=.0
      DO 220 I=1,NEQ
      S=DCMPLX(.0D0,.0D0)
      DO 210 J=1,NEQ
  210 S=S+Z(I,J)*V(J)
      SA=CDABS(S)
      IF(SA.GT.CMX)CMX=SA
  220 C(I)=S
      IF(IWR.LE.0)GO TO 250
      WRITE(17,5)
      DO 240 I=1,NEQ
      S=C(I)
      SA=CDABS(S)
      SN=SA/CMX
      PH=.0
      IF(SA.LE..0)GO TO 240
      PH=57.29578*DATAN2(DIMAG(S),DREAL(S))
  240 WRITE(17,2)I,SN,SA,PH
      WRITE(17,5)
  250 RETURN
      END
C
```

```
C                                    10
C
      SUBROUTINE DZ11(AK,BAR,DKD,DKW,EC,FB,HDK,TK,IFB,D11,DV1)          DZ11.1
C     DZ11 CALCULATES D11 = CHANGE IN SELF-IMPEDANCE OF MODE 1
C     DUE TO REFLECTION FROM FLAT EARTH.
C     ALSO DV1 = ONE TERM IN VOLTAGE FOR MODE 1.
      IMPLICIT REAL*8 (A-H), (P-Z)
      DIMENSION FB(1)
      COMPLEX*16 FST,G,GAM,RC,EC,ZAA,ZHH,PC,EG1,EG2
      COMPLEX*16 ZDD,ZMM,EST,ERD,ERM,EGZ,CB,D11,ZDM,ZMD
      COMPLEX*16 DV1,VDD,VDW,VHH,VAA
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      BAL=DLOG(BAR)
      BK=BAR*AK
      RH2=AK+DKD
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      SDKD=DSIN(DKD)
      DK1=DKW+TK
      SDK1=DSIN(DK1)
      CDK1=DCOS(DK1)
      EST=DCMPLX(.0D0,-ETA/(4.*PI*SDK1))
      FST=DCMPLX(.0D0,-ETA/(4.*PI*SDKD))
      DBET=.1
      KMX=200
      IF(KMX.GT.IFB)KMX=IFB
C  NEXT CALCULATE F(BETA) BY INTEGRATING ACROSS THE DISK.
      DO 60 K=1,KMX
      DRK=PI/10.
      BET=DBET*(K-1)
      IF(BET.GT.1.)DRK=DRK/BET
      INT=(RH2-AK)/DRK
      IF(INT.LT.10)INT=10
      DRK=(RH2-AK)/INT
      F=.0
      RK=AK+DRK/2.
C  NEXT INTEGRATE ACROSS THE DISK.
      DO 50 I=1,INT
      BR=BET*RK
      CALL BES10  (BR,BJ0,BJ1,0)
      F=F+BJ0*DCOS(RH2-RK)
   50 RK=RK+DRK
      FB(K)=DRK*F
   60 CONTINUE
C  NEXT CALCULATE ZDD.
      ZK=HDK+TK
      Z1=HDK
      Z2=HDK+TK+DKW
      ERD=DCMPLX(.0D0,.0D0)
      ERM=DCMPLX(.0D0,.0D0)
      VDD=DCMPLX(.0D0,.0D0)
C  NEXT INTEGRATE ON BETA.
      DO 80 K=1,KMX
      BET=DBET*(K-1)
      F=FB(K)
      CALL BES10   (BET*AK,BA0,BA1,0)
      BETS=BET*BET
      IF(BET.GT.1.)GO TO 62
      HR=DSQRT(1.-BETS)
      ARG=HR*ZK
      EGZ=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z1
      EG1=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z2
      EG2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HR)
```

```
      GO TO 64
  62  ALP=DSQRT(BETS-1.)
      EG2=DCMPLX(.0D0,.0D0)
      ARG=ALP*Z2
      IF(ARG.LT.80.)EG2=DCMPLX(DEXP(-ARG),.0D0)
      ARG=ALP*ZK
      IF(ARG.GT.80.)GO TO 81
      EGZ=DCMPLX(DEXP(-ARG),.0D0)
      EG1=DCMPLX(DEXP(-ALP*Z1),.0D0)
      GAM=DCMPLX(ALP,.0D0)
  64  G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
      CB=EG2-CDK1*EG1
C  NEXT INTEGRATE ON RHO.
      RSJ=.0
      DRK=PI/10.
      IF(BET.GT.1.)DRK=DRK/BET
      INT=DKD/DRK
      IF(INT.LT.10)INT=10
      DRK=DKD/INT
      RK=AK+DRK/2.
      DO 70 I=1,INT
      CALL BES10  (BET*RK,BJ0,BJ1,1)
      RSJ=RSJ+BJ1*DSIN(RH2-RK)
  70  RK=RK+DRK
      RSJ=DRK*RSJ
      ERD=ERD+RSJ*GAM*RC*F*EG1*EGZ
      ERM=ERM+RSJ*RC*BA0*CB*EGZ
      CALL BES10  (BET*BK,BB0,BB1,0)
      VDD=VDD+RSJ*RC*(BA0-BB0)*EGZ*EGZ
  80  CONTINUE
  81  ERD=DBET*ERD*DCMPLX(.0D0,-ETA/(4.*PI*SDKD*SDKD))
      ERM=DBET*ERM*DCMPLX(.0D0,-ETA/(4.*PI*SDKD*SDK))
      ZDD=ERD+ERM
      VDD=-DBET*VDD/(2.*BAL*SDKD)
C  NEXT CALCULATE ZDM BY INTEGRATING ON BETA.
      Z1=HDK+TK
      Z2=Z1+DKW
      ZDM=DCMPLX(.0D0,.0D0)
      DO 90 K=1,KMX
      BET=DBET*(K-1)
      F=FB(K)
      BA=BET*AK
      CALL BES10  (BA,BJ0,BJ1,0)
      BETS=BET*BET
      IF(BET.GT.1.)GO TO 82
      HR=DSQRT(1.-BETS)
      ARG=HR*HDK
      EGZ=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z1
      EG1=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z2
      EG2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HR)
      GO TO 84
  82  ALP=DSQRT(BETS-1.)
      EG2=DCMPLX(.0D0,.0D0)
      ARG=ALP*Z2
      IF(ARG.LT.80.)EG2=DCMPLX(DEXP(-ARG),.0D0)
      ARG=ALP*Z1
      IF(ARG.GT.80.)GO TO 92
      EG1=DCMPLX(DEXP(-ARG),.0D0)
      EGZ=DCMPLX(DEXP(-ALP*HDK),.0D0)
      GAM=DCMPLX(ALP,.0D0)
  84  G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
```

```
      IF(K.GT.1)GO TO 86
      PC=(DCMPLX(1.D0,2.D0*DKW)*EG2-DCMPLX(CDK,SDK)*EG1)/4.
      GO TO 90
   86 PC=((GAM*SDK-CDK)*EG1+EG2)/BETS
   90 ZDM=ZDM+BET*RC*F*BJ0*EGZ*PC
   92 ZDM=-DBET*FST*ZDM/SDK
C  NEXT CALCULATE ZHH BY INTEGRATING ON H.
      VHH=DCMPLX(.0D0,.0D0)
      ZHH=DCMPLX(.0D0,.0D0)
      DH=DBET
      NS=1./DH
      DH=1./NS
      HR=DH/2.
      DO 100 I=1,NS
      BETS=1.-HR*HR
      BET=DSQRT(BETS)
      CALL BES10  (BET*AK,BA0,BA1,0)
      CALL BES10  (BET*BK,BB0,BB1,0)
      GAM=DCMPLX(.0D0,HR)
      G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
      EGZ=DCMPLX(DCOS(HR*HDK),-DSIN(HR*HDK))
      EG1=DCMPLX(DCOS(HR*Z1),-DSIN(HR*Z1))
      EG2=DCMPLX(DCOS(HR*Z2),-DSIN(HR*Z2))
      PC=((GAM*SDK-CDK)*EG1+EG2)/BETS
      HR=HR+DH
      VHH=VHH+RC*(BA0-BB0)*BA0*EG1*PC
  100 ZHH=ZHH+RC*BA0*BA0*(EG2-CDK1*EGZ)*PC
      VHH=DH*VHH*DCMPLX(.0D0,-1.D0/(2.D0*BAL*SDK))
      ZHH=DH*ZHH*ETA/(4.*PI*SDK*SDK1)
C  NEXT CALCULATE ZAA BY INTEGRATING ON ALP.
      DA=DBET
      ALP=DA/2.
      VAA=DCMPLX(.0D0,.0D0)
      ZAA=DCMPLX(.0D0,.0D0)
      DO 110 I=1,KMX
      BETS=ALP*ALP+1.
      BET=DSQRT(BETS)
      CALL BES10  (BET*AK,BA0,BA1,0)
      CALL BES10  (BET*BK,BB0,BB1,0)
      G=CDSQRT(BETS-EC)
      RC=(ALP*EC-G)/(ALP*EC+G)
      EA2=.0
      ARG=ALP*Z2
      IF(ARG.LT.80.)EA2=DEXP(-ARG)
      ARG=ALP*Z1
      IF(ARG.GT.80.)GO TO 112
      EA1=DEXP(-ARG)
      EAZ=DEXP(-ALP*HDK)
      P=((ALP*SDK-CDK)*EA1+EA2)/BETS
      VAA=VAA+RC*(BA0-BB0)*BA0*EA1*P
      ZAA=ZAA+RC*BA0*BA0*(EA2-CDK1*EAZ)*P
  110 ALP=ALP+DA
  112 ZAA=-DA*EST*ZAA/SDK
      VAA=DA*VAA/(2.*BAL*SDK)
      ZMD=ZAA+ZHH
      ZMM=ZDM+ZMD
      DV1=VAA+VDD+VHH
      D11=ZDD+ZMM
      RETURN
      END
C
C
```

```
C                                       13
C
      SUBROUTINE DZDD(AK,DBET,DKD,DKW,EC,FB,HDK,S2,T2,TK          DZDD.1
     2,IFB,I12,KMX,D12,D22)
C     DZDD CALCULATES D12 = CHANGE IN MUTUAL IMPEDANCE BETWEEN MODE 1
C     AND DISK DIPOLE MODE.
C     ALSO D22 = CHANGE IN MUTUAL IMPEDANCE BETWEEN TWO DISK DIPOLE MODES.
      IMPLICIT REAL*8 (A-H), (P-Z)
      DIMENSION FB(1)
      COMPLEX*16 D12,D22,EC,QST,GAM,G,RC,EGZ,ZDD,ZDW,EZD,EZ1,EZ2,PC,QDW
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      SDKD=DSIN(DKD)
      CDKD=DCOS(DKD)
      S1=S2-DKD
      S3=S2+DKD
      T1=T2-DKD
      T3=T2+DKD
      QST=DCMPLX(.0D0,-ETA/(4.*PI*SDKD*SDKD))
      IF(I12.GT.1)GO TO 62
      DBET=.1
      KMX=200
      IF(KMX.GT.IFB)KMX=IFB
C  NEXT CALCULATE F(BETA) BY INTEGRATING ACROSS THE ANNULAR DISK.
      DO 60 K=1,KMX
      DRK=PI/10.
      BET=DBET*(K-1)
      IF(BET.GT.1.)DRK=DRK/BET
      INT=DKD/DRK
      IF(INT.LT.10)INT=10
      DRK=DKD/INT
      F=.0
      RK=S1+DRK/2.
C  NEXT INTEGRATE ACROSS THE DISK.
      DO 50 L=1,2
      DO 40 I=1,INT
      CALL BES10   (BET*RK,BJ0,BJ1,0)
      IF(L.EQ.1)F=F-BJ0*DCOS(RK-S1)
      IF(L.EQ.2)F=F+BJ0*DCOS(S3-RK)
   40 RK=RK+DRK
   50 RK=S2+DRK/2.
      FB(K)=DRK*F
   60 CONTINUE
   62 CONTINUE
C  NEXT CALCULATE D22.
      ZK=2.*HDK+TK
C  NEXT INTEGRATE ON BETA.
      D22=DCMPLX(.0D0,.0D0)
      DO 100 K=1,KMX
      DRK=PI/10.
      BET=DBET*(K-1)
      IF(BET.GT.1.)DRK=DRK/BET
      INT=DKD/DRK
      IF(INT.LT.10)INT=10
      DRK=DKD/INT
      F=FB(K)
      BETS=BET*BET
      IF(BET.GT.1.)GO TO 72
      HR=DSQRT(1.-BETS)
      ARG=HR*ZK
      EGZ=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HR)
      GO TO 74
   72 ALP=DSQRT(BETS-1.)
      ARG=ALP*ZK
      IF(ARG.GT.80.)GO TO 102
      EGZ=DCMPLX(DEXP(-ARG),.0D0)
```

```
      GAM=DCMPLX(ALP,.0D0)
   74 G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
C  NEXT INTEGRATE ACROSS THE ANNULAR DISK.
      R22=.0
      RK=T1+DRK/2.
      DO 90 L=1,2
      DO 80 I=1,INT
      CALL BES10  (BET*RK,BJ0,BJ1,1)
      IF(L.EQ.1)R22=R22+BJ1*DSIN(RK-T1)
      IF(L.EQ.2)R22=R22+BJ1*DSIN(T3-RK)
   80 RK=RK+DRK
   90 RK=T2+DRK/2.
  100 D22=D22+DRK*GAM*RC*F*EGZ*R22
  102 D22=DBET*QST*D22
      IF(I12.NE.1)RETURN
C  NEXT CALCULATE D12.
      ZDD=DCMPLX(.0D0,.0D0)
      ZDW=DCMPLX(.0D0,.0D0)
      R2=AK+DKD
C  NEXT INTEGRATE ON BETA.
      Z1=HDK+TK
      Z2=Z1+DKW
      SDK=DSIN(DKW)
      CDK=DCOS(DKW)
      QDW=DCMPLX(.0D0,-ETA/(4.*PI*SDK*SDKD))
      DO 160 K=1,KMX
      DRK=PI/10.
      BET=DBET*(K-1)
      IF(BET.GT.1.)DRK=DRK/BET
      INT=DKD/DRK
      IF(INT.LT.10)INT=10
      DRK=DKD/INT
      F=FB(K)
      BETS=BET*BET
      IF(BET.GT.1.)GO TO 112
      HR=DSQRT(1.-BETS)
      ARG=HR*ZK
      EGZ=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*HDK
      EZD=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z1
      EZ1=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z2
      EZ2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HR)
      GO TO 114
  112 ALP=DSQRT(BETS-1.)
      EZ2=DCMPLX(.0D0,.0D0)
      ARG=ALP*Z2
      IF(ARG.LT.80.)EZ2=DCMPLX(DEXP(-ARG),.0D0)
      EGZ=DCMPLX(.0D0,.0D0)
      ARG=ALP*ZK
      IF(ARG.LT.80.)EGZ=DCMPLX(DEXP(-ARG),.0D0)
      ARG=ALP*Z1
      IF(ARG.GT.80.)GO TO 162
      EZ1=DCMPLX(DEXP(-ARG),.0D0)
      EZD=DCMPLX(DEXP(-ALP*HDK),.0D0)
      GAM=DCMPLX(ALP,.0D0)
  114 G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
C  NEXT INTEGRATE ACROSS THE DISK.
      R22=.0
      RK=AK+DRK/2.
      DO 140 I=1,INT
      CALL BES10  (BET*RK,BJ0,BJ1,1)
```

```
      R22=R22+BJ1*DSIN(RK-R2)                                        DZDD.3
  140 RK=RK+DRK
      CALL BES10   (BET*AK,BJ0,BJ1,0)
      IF(K.GT.1)GO TO 150
      PC=(DCMPLX(1.D0,2.*DKW)*EZ2-DCMPLX(CDK,SDK)*EZ1)/4.
      GO TO 152
  150 PC=((GAM*SDK-CDK)*EZ1+EZ2)/BETS
  152 ZDW=ZDW+BET*RC*F*BJ0*EZD*PC
  160 ZDD=ZDD+DRK*GAM*RC*F*EGZ*R22
  162 ZDD=DBET*QST*ZDD
      ZDW=DBET*QDW*ZDW
      D12=ZDD+ZDW
      RETURN
      END
C
C
```

```
C
      SUBROUTINE DZWD (AK,DKD,DKW,EC,HDK,NSD,TK,ZK2,DZ12)             DZWD.1
C     DZWD  CALCULATES DZ12 =  CHANGE IN MUTUAL IMPEDANCE
C     BETWEEN A WIRE DIPOLE MODE AND A DISK DIPOLE MODE.
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 DZ12(1),EC,GAM,G,RC,QC,EGZ,QST
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      IF(NSD.LE.1)RETURN
      DO 20 I=2,NSD
   20 DZ12(I)=DCMPLX(.0D0,.0D0)
      TDKD=2.*DKD
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      SDKD=DSIN(DKD)
      Z1=HDK+TK
      ZJ2=Z1+ZK2
      ZT=Z1+ZJ2
      DBET=.1
      KMX=200
      BET=DBET/2.
C  NEXT INTEGRATE ON BETA.
      DO 100 K=1,KMX
      BETS=BET*BET
      IF(BET.GT.1.)GO TO 42
      HR=DSQRT(1.-BETS)
      GAM=DCMPLX(.0D0,HR)
      CGD=DCOS(HR*DKW)
      ARG=HR*ZT
      EGZ=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GO TO 44
   42 ALP=DSQRT(BETS-1.)
      GAM=DCMPLX(ALP,.0D0)
      EAD=DEXP(ALP*DKW)
      CGD=(EAD+1./EAD)/2.
      ARG=ALP*ZT
      IF(ARG.GT.80.)GO TO 102
      EAZ=DEXP(-ARG)
      EGZ=DCMPLX(EAZ,.0D0)
   44 G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
      CALL BES10  (BET*AK,BJ0,BJ1,0)
      CC=CGD-CDK
      DRK=PI/10.
      IF(BET.GT.1.)DRK=DRK/BET
      INT=DKD/DRK
      IF(INT.LT.10)INT=10
      DRK=DKD/INT
      RH1=AK
      QC=(RC*EGZ)*(DRK*BJ0*CC)
      DO 80 I=2,NSD
      RH2=RH1+DKD
      RH3=RH1+TDKD
C  NEXT INTEGRATE ACROSS THE ANNULAR DISK.
      FR=.0
      RK=RH1+DRK/2.
      DO 70 L=1,2
      DO 60 J=1,INT
      CALL BES10  (BET*RK,BJ0,BJ1,1)
      IF(L.EQ.1)CI=DSIN(RK-RH1)
      IF(L.EQ.2)CI=DSIN(RH3-RK)
      FR=FR+BJ1*CI
   60 RK=RK+DRK
   70 RK=RH2+DRK/2.
      RH1=RH1+DKD
   80 DZ12(I)=DZ12(I)+QC*FR
  100 BET=BET+DBET
```

17

```
  102 QST=DCMPLX(.0D0,DBET*ETA/(TP*SDKD*SDK))                          DZWD.2
      DO 120 I=2,NSD
  120 DZ12(I)=QST*DZ12(I)
      RETURN
      END
C
C
```

```
C                                  18
      SUBROUTINE DZWW(AK,DKD,DKW,EC,HDK,J,TK,DZIJ,DZ1J)               DZWW.1
C     DZWW CALCULATES DZIJ = CHANGE IN MUTUAL IMPEDANCE OF
C     TWO WIRE DIPOLE MODES.
C     AND DZ1J = CHANGE IN MUTUAL IMPEDANCE BETWEEN MODE 1 AND
C     A WIRE DIPOLE MODE.
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 DZIJ(1),DZ1J
      COMPLEX*16 G,GAM,EC,RC,QC,QJ,EJH,EIH,EZ1,EZ2,PC,ZDW,EGZ
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      QJ=DCMPLX(.0D0,1.D0)
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      SDKD=DSIN(DKD)
      DO 20 I=1,J
   20 DZIJ(I)=DCMPLX(.0D0,.0D0)
      DZ1J=DCMPLX(.0D0,.0D0)
      Z1=HDK+TK
      Z2=Z1+DKW
      ZJ2=Z1+J*DKW
C  NEXT INTEGRATE ON H.
C     DH=.25/ZJ2
      DH=.1
      KMX=1./DH
      IF(KMX.LT.10)KMX=10
      DH=1./KMX
      HR=DH/2.
      DO 100 K=1,KMX
      BETS=1.-HR*HR
      BET=DSQRT(BETS)
      CALL BES10  (BET*AK,BJ0,BJ1,0)
      GAM=DCMPLX(.0D0,HR)
      G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
      CC=DCOS(HR*DKW)-CDK
      FAC=BJ0*CC
      QC=RC*(FAC*FAC*DH/BETS)
      ARG=HR*ZJ2
      EJH=DCMPLX(DCOS(ARG),-DSIN(ARG))
      QC=QC*EJH
      ZI2=Z1
      DO 80 I=1,J
      ZI2=ZI2+DKW
      ARG=HR*ZI2
      EIH=DCMPLX(DCOS(ARG),-DSIN(ARG))
   80 DZIJ(I)=DZIJ(I)+QC*EIH
      BJ2=BJ0*BJ0/BETS
      ARG=HR*Z1
      EZ1=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z2
      EZ2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      PC=EZ2+EZ1*DCMPLX(-CDK,HR*SDK)
      DZ1J=DZ1J+RC*BJ2*CC*EJH*PC*DH
  100 HR=HR+DH
C  NEXT INTEGRATE ON ALP.
C     AMX=6./(ZJ2+Z1)
C     DA=.1/ZJ2
C     KMX=AMX/DA
      DA=DH
      KMX=200
      ALP=DA/2.
      DO 200 K=1,KMX
      BETS=ALP*ALP+1.
      BET=DSQRT(BETS)
      CALL BES10  (BET*AK,BJ0,BJ1,0)
      G=CDSQRT(BETS-EC)
```

```
      RC=(ALP*EC-G)/(ALP*EC+G)
      EAD=DEXP(ALP*DKW)
      CAD=(EAD+1./EAD)/2.
      CC=CAD-CDK
      FAC=BJ0*CC
      QC=(QJ*RC)*(FAC*FAC*DA/BETS)
      ARG=ALP*ZJ2
      IF(ARG.GT.80.)GO TO 202
      AJZ=DEXP(-ARG)
      QC=QC*AJZ
      ZI2=Z1
      DO 180 I=1,J
      ZI2=ZI2+DKW
      AIZ=DEXP(-ALP*ZI2)
  180 DZIJ(I)=DZIJ(I)+QC*AIZ
      BJ2=BJ0*BJ0/BETS
      AZ1=DEXP(-ALP*Z1)
      AZ2=DEXP(-ALP*Z2)
      PR=AZ2+(ALP*SDK-CDK)*AZ1
      DZ1J=DZ1J+QJ*RC*BJ2*CC*AJZ*PR*DA
  200 ALP=ALP+DA
  202 RIJ=ETA/(PI*SDK*SDK)
      DZ1J=RIJ*DZ1J/2.
      DO 220 I=1,J
  220 DZIJ(I)=RIJ*DZIJ(I)
C  NEXT INTEGRATE ON BETA.
      KMX=200
      ZT=Z1+ZJ2
      RH2=AK+DKD
      ZDW=DCMPLX(.0D0,.0D0)
      DBET=.1
      BET=DBET/2.
      DO 300 K=1,KMX
      BETS=BET*BET
      IF(BET.GT.1.)GO TO 262
      HR=DSQRT(1.-BETS)
      ARG=HR*ZT
      EGZ=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HR)
      CGD=DCOS(HR*DKW)
      GO TO 264
  262 ALP=DSQRT(BETS-1.)
      ARG=ALP*ZT
      IF(ARG.GT.80.)GO TO 302
      AGZ=DEXP(-ARG)
      EGZ=DCMPLX(AGZ,.0D0)
      GAM=DCMPLX(ALP,.0D0)
      EAD=DEXP(ALP*DKW)
      CGD=(EAD+1./EAD)/2.
  264 G=CDSQRT(BETS-EC)
      RC=(GAM*EC-G)/(GAM*EC+G)
      CALL BES10   (BET*AK,BJ0,BJ1,0)
      CC=CGD-CDK
      QC=RC*BJ0*CC*EGZ
      DRK=PI/10.
      IF(BET.GT.1.)DRK=DRK/BET
      INT=DKD/DRK
      IF(INT.LT.10)INT=10
      DRK=DKD/INT
      RK=AK+DRK/2.
      RDW=.0
C  NEXT INTEGRATE ON RHO.
      DO 280 I=1,INT
      CALL BES10   (BET*RK,BJ0,BJ1,1)
      RDW=RDW+BJ1*DSIN(RH2-RK)
  280 RK=RK+DRK
```

```
                                     20
      ZDW=ZDW+DRK*QC*RDW                                                DZWW.3
  300 BET=BET+DBET
  302 ZDW=DBET*ZDW*DCMPLX(.0D0,-ETA/(TP*SDKD*SDK))
      DZ1J=DZ1J+ZDW
      RETURN
      END
C
C
```

```
C
C                                        21
      SUBROUTINE EXPJ(V1,V2,W12)                                        EXPJ.1
C     EXPJ CALCULATES W12 = EXPONENTIAL INTEGRAL WITH
C     LOWER LIMIT V1 AND UPPER LIMIT V2.
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 EC,E15,S,T,UC,VC,V1,V2,W12,Z
      DIMENSION V(21),W(21),D(16),E(16)
      DATA V/   0.22284667D 00,
     20.11889321D 01,0.29927363D 01,0.57751436D 01,0.98374674D 01,
     20.15982874D 02,0.93307812D-01,0.49269174D 00,0.12155954D 01,
     20.22699495D 01,0.36676227D 01,0.54253366D 01,0.75659162D 01,
     20.10120228D 02,0.13130282D 02,0.16654408D 02,0.20776479D 02,
     20.25623894D 02,0.31407519D 02,0.38530683D 02,0.48026086D 02/
      DATA W/   0.45896460D 00,
     20.41700083D 00,0.11337338D 00,0.10399197D-01,0.26101720D-03,
     20.89854791D-06,0.21823487D 00,0.34221017D 00,0.26302758D 00,
     20.12642582D 00,0.40206865D-01,0.85638778D-02,0.12124361D-02,
     20.11167440D-03,0.64599267D-05,0.22263169D-06,0.42274304D-08,
     20.39218973D-10,0.14565152D-12,0.14830270D-15,0.16005949D-19/
      DATA D/   0.22495842D 02,
     2 0.74411568D 02,-0.41431576D 03,-0.78754339D 02, 0.11254744D 02,
     2 0.16021761D 03,-0.23862195D 03,-0.50094687D 03,-0.68487854D 02,
     2 0.12254778D 02,-0.10161976D 02,-0.47219591D 01, 0.79729681D 01,
     2-0.21069574D 02, 0.22046490D 01, 0.89728244D 01/
      DATA E/   0.21103107D 02,
     2-0.37959787D 03,-0.97489220D 02, 0.12900672D 03, 0.17949226D 02,
     2-0.12910931D 03,-0.55705574D 03, 0.13524801D 02, 0.14696721D 03,
     2 0.17949528D 02,-0.32981014D 00, 0.31028836D 02, 0.81657657D 01,
     2 0.22236961D 02, 0.39124892D 02, 0.81636799D 01/
      Z=V1
      DO 100 JIM=1,2
      X=DREAL(Z)
      Y=DIMAG(Z)
      E15=DCMPLX(.0D0,.0D0)
      AB=CDABS(Z)
      IF(AB.EQ.0.)GO TO 90
      IF(X.GE.0.  .AND. AB.GT.10.)GO TO 80
      YA=DABS(Y)
      IF(X.LE.0.  .AND. YA.GT.10.)GO TO 80
      IF(YA-X.GE.17.5.OR.YA.GE.6.5.OR.X+YA.GE.5.5.OR.X.GE.3.)GO TO 20
      IF(X.LE.-9.)GO TO 40
      IF(YA-X.GE.2.5)GO TO 50
      IF(X+YA.GE.1.5)GO TO 30
   10 N=6.+3.*AB
      E15=1./(N-1.)-Z/N**2
   15 N=N-1
      E15=1./(N-1.)-Z*E15/N
      IF(N.GE.3)GO TO 15
      E15=Z*E15-DCMPLX(.577216+DLOG(AB),DATAN2(Y,X))
      GO TO 90
   20 J1=1
      J2=6
      GO TO 31
   30 J1=7
      J2=21
   31 S=DCMPLX(.0D0,.0D0)
      YS=Y*Y
      DO 32 I=J1,J2
      XI=V(I)+X
      CF=W(I)/(XI*XI+YS)
   32 S=S+DCMPLX(XI*CF,-YA*CF)
      GO TO 54
   40 T3=X*X-Y*Y
      T4=2.*X*YA
      T5=X*T3-YA*T4
```

```
      T6=X*T4+YA*T3
      UC=DCMPLX(D(11)+D(12)*X+D(13)*T3+T5-E(12)*YA-E(13)*T4,
     2 E(11)+E(12)*X+E(13)*T3+T6+D(12)*YA+D(13)*T4)
      VC=DCMPLX(D(14)+D(15)*X+D(16)*T3+T5-E(15)*YA-E(16)*T4,
     2 E(14)+E(15)*X+E(16)*T3+T6+D(15)*YA+D(16)*T4)
      GO TO 52
   50 T3=X*X-Y*Y
      T4=2.*X*YA
      T5=X*T3-YA*T4
      T6=X*T4+YA*T3
      T7=X*T5-YA*T6
      T8=X*T6+YA*T5
      T9=X*T7-YA*T8
      T10=X*T8+YA*T7
      UC=DCMPLX(D(1)+D(2)*X+D(3)*T3+D(4)*T5+D(5)*T7+T9-(E(2)*YA+E(3)*T4
     2+E(4)*T6+E(5)*T8),E(1)+E(2)*X+E(3)*T3+E(4)*T5+E(5)*T7+T10+
     3(D(2)*YA+D(3)*T4+D(4)*T6+D(5)*T8))
      VC=DCMPLX(D(6)+D(7)*X+D(8)*T3+D(9)*T5+D(10)*T7+T9-(E(7)*YA+E(8)*T4
     2+E(9)*T6+E(10)*T8),E(6)+E(7)*X+E(8)*T3+E(9)*T5+E(10)*T7+T10+
     3(D(7)*YA+D(8)*T4+D(9)*T6+D(10)*T8))
   52 EC=UC/VC
      S=EC/DCMPLX(X,YA)
   54 EX=DEXP(-X)
      T=EX*DCMPLX(DCOS(YA),-DSIN(YA))
      E15=S*T
   56 IF(Y.LT.0.)E15=DCONJG(E15)
      GO TO 90
   80 E15=.409319/(Z+.193044)+.421831/(Z+1.02666)+.147126/(Z+2.56788)+
     2.206335E-1/(Z+4.90035)+.107401E-2/(Z+8.18215)+.158654E-4/(Z+
     312.7342)+.317031E-7/(Z+19.3957)
      E15=E15*CDEXP(-Z)
   90 IF(JIM.EQ.1)W12=E15
  100 Z=V2
      Z=V2/V1
      TH=DATAN2(DIMAG(Z),DREAL(Z))-DATAN2(DIMAG(V2),DREAL(V2))
     2+DATAN2(DIMAG(V1),DREAL(V1))
      AB=DABS(TH)
      IF(AB.LT.1.)TH=.0
      IF(TH.GT.1.)TH=6.2831853
      IF(TH.LT.-1.)TH=-6.2831853
      W12=W12-E15+DCMPLX(.0D0,TH)
      RETURN
      END
C
C
```

```
C
C                                23
      SUBROUTINE GRILL(AK,BAR,DKD,DKW,NEQ,NSD,NSW,TK,VJ)               GRILL.1
C     GRILL CALCULATES THE VOLTAGE COLUMN VJ(I) FOR
C     MONOPOLE ON CIRCULAR DISK IN FREE SPACE.
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 EGZ,GM,GP,GI(20),VJ(1),GII,QST,WST,VJ1
      DATA PI,TP/3.14159265359,6.28318530718/
      IDM=20
      DO 20 I=1,NEQ
   20 VJ(I)=DCMPLX(.0D0,.0D0)
      VJ(1)=DCMPLX(1.D0,.0D0)
      IF(BAR.LE.1.)RETURN
      VJ(1)=DCMPLX(.0D0,.0D0)
      DK=DKW
      SDK=DSIN(DK)
      CDK=DCOS(DK)
      BAL=DLOG(BAR)
      QST=DCMPLX(.0D0,1./(4.*BAL*SDK))
      BK=AK*BAR
      AKS=AK*AK
      BKS=BK*BK
      LIM=NSW+1
      IF(LIM.GT.IDM)LIM=IDM
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      PH=.0
      DO 90 LPH=1,2
      WST=DPH*QST/(3.*PI)
      SGN=-1.
      DO 80 IPH=1,NPP
      WF=3.+SGN
      IF(IPH.EQ.1)WF=1.
      IF(IPH.EQ.NPP)WF=1.
      CPH=DCOS(PH)
      IF(IPH.GT.1)GO TO 40
      IF(LPH.GT.1)GO TO 40
      CPH=DCOS(DPH/10.)
   40 RS1=2.*AKS*(1.-CPH)
      RS2=AKS+BKS-2.*AK*BK*CPH
      RH1=DSQRT(RS1)
      RH2=DSQRT(RS2)
      CALL CISI (CA,CIN,SA,RH1)
      CALL CISI2(CB,CIN,SB,RH2)
      GI(1)=2.*DCMPLX(CB-CA,SA-SB)
      DO 50 I=2,LIM
      DZ=DK*(I-1)
      DZS=DZ*DZ
      RA=DSQRT(RS1+DZS)
      RB=DSQRT(RS2+DZS)
      CALL CISI (C1,CIN,S1,RA+DZ)
      CALL CISI (C2,CIN,S2,RB+DZ)
      GP=DCMPLX(C2-C1,S1-S2)
      RAM=RS1/(RA+DZ)
      RBM=RS2/(RB+DZ)
      CALL CISI (C1,CIN,S1,RAM)
      CALL CISI (C2,CIN,S2,RBM)
      GM=DCMPLX(C2-C1,S1-S2)
      EGZ=DCMPLX(DCOS(DZ),DSIN(DZ))
   50 GI(I)=GP*EGZ+GM/EGZ
      VJ(1)=VJ(1)+WF*WST*(GI(2)-CDK*GI(1))
      IF(NSW.LE.1)GO TO 78
      K1=0
```

```
      IA=NSD+1
      DO 60 I=IA,NEQ
      K1=K1+1
      K2=K1+1
      K3=K2+1
      IF(K3.GT.IDM)GO TO 60
      GP=GI(K1)-2.*CDK*GI(K2)+GI(K3)
      VJ(I)=VJ(I)+WF*WST*GP
   60 CONTINUE
   78 SGN=-SGN
   80 PH=PH+DPH
      DPH=(PI-PHA)/NPH
   90 PH=PHA
      CALL CISI (CA,CIN,SA,AK)
      CALL CISI (CB,CIN,SB,BK)
      R2=AK+DKD
      SR2=DSIN(R2)
      CR2=DCOS(R2)
      SDKD=DSIN(DKD)
      V11=(SR2*(CB-CA)-CR2*(SB-SA))/(2.*BAL*SDKD)
      VJ(1)=V11+VJ(1)
      IF(NSD.LE.1)RETURN
      V22=(DSIN(AK)*(CB-CA)-DCOS(AK)*(SB-SA))/(2.*BAL*SDKD)
      VJ(2)=DCMPLX(V22,.0D0)
      RETURN
      END
C
C
```

```
C
      SUBROUTINE QDD(CDKD,SDKD,S1,S3,T1,T3,TK,IWZ,NPH,Z22)          QDD.1
C     QDD CALCULATES Z22 = MUTUAL IMPEDANCE OF TWO DISK MODES.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 Z12,Z22,ZD
      DATA PI,P3/3.14159265359,9.42477796077/
    2 FORMAT(1X,8F10.2)
    5 FORMAT(1H0)
    6 FORMAT(5X,'PRINTOUT FROM QDD')
    7 FORMAT(5X,'DISK DIPOLE TO DISK DIPOLE')
      IF(IWZ.LE.0)GO TO 10
      WRITE(17,6)
      WRITE(17,7)
      WRITE(17,5)
   10 PHA=.0174533*2.
      PHB=.0174533*20.
      NPH=2*(NPH/2)
      DPH=PHA/NPH
      PH=.0
      NPHP=NPH+1
      Z22=(.0D0,.0D0)
      DO 80 IPH=1,3
      ZD=(.0D0,.0D0)
      SGI=-1.
      DO 70 I=1,NPHP
      WF=3.+SGI
      IF(I.EQ.1)WF=1.
      IF(I.EQ.NPHP)WF=1.
      CPH=DCOS(PH)
      IF(I.EQ.1 .AND. IPH.GT.1)GO TO 60
C  NEXT:  DISK DIPOLE TO DISK DIPOLE.
      CALL SKEWS (S1,S3,T1,T3,TK,CDKD,SDKD,CDKD,SDKD,CPH,Z12)
   60 PHD=57.29578*PH
      IF(IWZ.GT.0)WRITE(17,2)PHD,Z12
      SGI=-SGI
      PH=PH+DPH
   70 ZD=ZD+WF*Z12
      Z22=Z22+DPH*ZD/P3
      PH=PHA
      DPH=(PHB-PHA)/NPH
      IF(IPH.EQ.1)GO TO 80
      PH=PHB
      DPH=(PI-PHB)/NPH
   80 CONTINUE
      IF(IWZ.GT.0)WRITE(17,5)
      RETURN
      END
C
C
```

```
C                                  26
C
      SUBROUTINE QDM(AK,DKD,DKW,CDKD,SDKD,SDK,S1,S3,TK,IWZ,NPH,Z12)   QDM.1
C     QDM CALCULATES Z12 = MUTUAL IMPEDANCE OF DISK DIPOLE AND MODE 1.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 ZD,Z12,Z21,ZDM,ZM,P12,ZDD,PDM
      DATA PI,P3/3.14159265359,9.42477796077/
    2 FORMAT(1X,8F10.2)
    5 FORMAT(1H0)
    6 FORMAT(5X,'PRINTOUT FROM QDM')
    7 FORMAT(5X,'DISK DIPOLE TO DISK MONOPOLE')
    8 FORMAT(5X,'DISK DIPOLE TO WIRE MONOPOLE')
      IF(IWZ.LE.0)GO TO 10
      WRITE(17,6)
      WRITE(17,7)
      WRITE(17,5)
   10 PHA=.0174533*2.
      PHB=.0174533*20.
      NPH=2*(NPH/2)
      NPHP=NPH+1
      DPH=PHA/NPH
      IDM=1
      IF(S1.GT.10.*AK)IDM=0
      PH=.0
      ZDD=(.0D0,.0D0)
      T2=AK+DKD
      DO 40 IPH=1,3
      ZD=(.0D0,.0D0)
      SGI=-1.
      DO 30 I=1,NPHP
      WF=3.+SGI
      IF(I.EQ.1)WF=1.
      IF(I.EQ.NPHP)WF=1.
      CPH=DCOS(PH)
      IF(I.EQ.1 .AND. IPH.GT.1)GO TO 20
C  NEXT:  DISK-DIPOLE TO DISK-MONOPOLE.
      CALL ZSDM (S1,S3,AK,T2,TK,CDKD,SDKD,SDKD,CPH,P12)
   20 PHD=57.29578*PH
      IF(IWZ.GT.0)WRITE(17,2)PHD,P12
      SGI=-SGI
      PH=PH+DPH
   30 ZD=ZD+WF*P12
      ZDD=ZDD+DPH*ZD/P3
      PH=PHA
      DPH=(PHB-PHA)/NPH
      IF(IPH.EQ.1)GO TO 40
      PH=PHB
      DPH=(PI-PHB)/NPH
   40 CONTINUE
      IF(IWZ.GT.0)WRITE(17,5)
      W2=TK+DKW
      IF(IDM.EQ.0)GO TO 100
      IF(IWZ.GT.0)WRITE(17,8)
      LPH=6
      LPH=2*(LPH/2)
      LPP=LPH+1
      PHA=.0174533*20.
      DPH=PHA/LPH
      PH=.0
      ZDM=(.0D0,.0D0)
      RMN=AK/100.
      DO 90 IPH=1,2
      PDM=(.0D0,.0D0)
      SGI=-1.
      DO 80 I=1,LPP
```

```
      WF=3.+SGI
      IF(I.EQ.1)WF=1.
      IF(I.EQ.LPP)WF=1.
      CPH=DCOS(PH)
      RH=AK*DSIN(PH)
      IF(I.EQ.1)RH=RMN
      AC=AK*CPH
      V1=S1-AC
      V3=S3-AC
      IF(I.EQ.1 .AND. IPH.GT.1)GO TO 70
C  NEXT:  DISK-DIPOLE TO WIRE-MONOPOLE.
      CALL ZSDM (V1,V3,TK,W2,RH,CDKD,SDKD,SDK,.0D0,P12)
   70 PHD=57.29578*PH
      IF(IWZ.GT.0)WRITE(17,2)PHD,P12
      SGI=-SGI
      PH=PH+DPH
   80 PDM=PDM+WF*P12
      ZDM=ZDM+DPH*PDM/P3
      DPH=(PI-PHA)/LPH
   90 PH=PHA
      Z12=ZDM-ZDD
      IF(IWZ.GT.0)WRITE(17,5)
      RETURN
C  NEXT:  DISK-DIPOLE TO WIRE-MONOPOLE.
  100 CALL ZSDM (S1,S3,TK,W2,AK,CDKD,SDKD,SDK,.0D0,ZDM)
      Z12=ZDM-ZDD
      RETURN
      END
C
C
```

```
C                                     28
C
      SUBROUTINE QMM(AK,DKD,DKW,CDKD,SDKD,CDK,SDK,TK,IWZ,NPH,Z11)      QMM.1
C     QMM CALCULATES Z11 = SELF IMPEDANCE OF MODE 1,
C     WHICH HAS TERMINALS AT BASE OF MONOPOLE.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 FDM,FMD,FMM,PDM,PMD,PMM
      COMPLEX*16 ZD,ZDD,ZDM,ZMD,ZMM,Z11,P11
      DATA PI,P3/3.14159265359,9.42477796077/
  1   FORMAT(1X,8F8.0)
  2   FORMAT(1X,8F10.2)
  5   FORMAT(1H0)
  6   FORMAT(5X,'PRINTOUT FROM QMM')
  7   FORMAT(5X,'DISK MONOPOLE TO DISK MONOPOLE')
      IF(IWZ.LE.0)GO TO 10
      WRITE(17,6)
      WRITE(17,5)
  10  AKS=AK*AK
      DKMP=TK+DKW
      CDKMP=DCOS(DKMP)
      SDKMP=DSIN(DKMP)
      ZMM=(.0D0,.0D0)
      ZMD=(.0D0,.0D0)
      ZDM=(.0D0,.0D0)
      LPH=6
      LPH=2*(LPH/2)
      LPP=LPH+1
      PHA=.0174533*20.
      DPH=PHA/LPH
      RMN=AK*DPH/10.
      PH=.0
      DO 44 IPH=1,2
      FDM=(.0D0,.0D0)
      FMD=(.0D0,.0D0)
      FMM=(.0D0,.0D0)
      SGI=-1.
      DO 40 I=1,LPP
      WF=3.+SGI
      IF(I.EQ.1)WF=1.
      IF(I.EQ.LPP)WF=1.
      IF(I.EQ.1 .AND. IPH.GT.1)GO TO 38
      CPH=DCOS(PH)
      SPH=DSIN(PH)
      DRG=2.*(1.-CPH)
      R=AK*DSQRT(DRG)
      IF(I.EQ.1)R=R+RMN
C  NEXT:  WIRE MONOPOLE TO WIRE MONOPOLE.
      CALL ZSMM (-TK,DKW,.0D0,DKW,R,CDKMP,SDKMP,SDK,1.D0,PMM)
      R=AK*SPH
      IF(I.EQ.1)R=R+RMN
      T1=AK*(1.-CPH)
      T2=T1+DKD
C  NEXT:  WIRE MONOPOLE TO DISK MONOPOLE.
      CALL ZSMM (-TK,DKW,T1,T2,R,CDKMP,SDKMP,SDKD,.0D0,PMD)
      S1=AK*(1.-CPH)
      S2=S1+DKD
C  NEXT:  DISK MONOPOLE TO WIRE MONOPOLE.
      CALL ZSMM (S1,S2,TK,TK+DKW,R,CDKD,SDKD,SDK,.0D0,PDM)
  38  FMM=FMM+WF*PMM
      FMD=FMD+WF*PMD
      FDM=FDM+WF*PDM
      PHD=57.29578*PH
      IF(IWZ.GT.0)WRITE(17,1)PHD,PMM,PMD,PDM
      SGI=-SGI
```

```
 40  PH=PH+DPH
     ZMM=ZMM+DPH*FMM/P3
     ZMD=ZMD+DPH*FMD/P3
     ZDM=ZDM+DPH*FDM/P3
     RMN=.0
     DPH=(PI-PHA)/LPH
 44  PH=PHA
     IF(IWZ.GT.0)WRITE(17,5)
     IF(IWZ.GT.0)WRITE(17,7)
     PHA=.0174533*2.
     PHB=.0174533*20.
     NPH=2*(NPH/2)
     NPHP=NPH+1
     DPH=PHA/NPH
     PH=.0
     ZDD=(.0D0,.0D0)
     S2=AK+DKD
     DO 60 IPH=1,3
     ZD=(.0D0,.0D0)
     SGI=-1.
     DO 50 I=1,NPHP
     WF=3.+SGI
     IF(I.EQ.1)WF=1.
     IF(I.EQ.NPHP)WF=1.
     CPH=DCOS(PH)
     IF(I.EQ.1 .AND. IPH.GT.1)GO TO 48
C  NEXT:  DISK MONOPOLE TO DISK MONOPOLE.
     CALL ZSMM (AK,S2,AK,S2,TK,CDKD,SDKD,SDKD,CPH,P11)
 48  PHD=57.29578*PH
     IF(IWZ.GT.0)WRITE(17,2)PHD,P11
     SGI=-SGI
     PH=PH+DPH
 50  ZD=ZD+WF*P11
     ZDD=ZDD+DPH*ZD/P3
     PH=PHA
     DPH=(PHB-PHA)/NPH
     IF(IPH.EQ.1)GO TO 60
     PH=PHB
     DPH=(PI-PHB)/NPH
 60  CONTINUE
     Z11=ZDD-ZDM-ZMD+ZMM
     IF(IWZ.GT.0)WRITE(17,5)
     RETURN
     END
C
C
```

```
C
C
      SUBROUTINE SKEW(S1,S3,T1,T3,RHK,CDK,SDK,CDKT,SDKT,CPSI,Z12)         SKEW.1
C     SKEW CALCULATES Z12 = MUTUAL IMPEDANCE OF CENTER-FED
C     NONPLANAR-SKEW SINUSOIDAL DIPOLES WITH UNEQUAL LENGTH.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 Z12,EIN,EGDZ,CQX,EJXX,EM,EP
      COMPLEX*16 E(2,2),F(2,2),ES1,ES2,ET1,ET2,EXPA,EXPB,EGZI
      COMPLEX*16 P11,P12,P21,P22
      DIMENSION S(3),T(3)
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      S(1)=S1
      S2=(S1+S3)/2.
      S(2)=S2
      S(3)=S3
      T(1)=T1
      T2=(T1+T3)/2.
      T(2)=T2
      T(3)=T3
      Z12=(.0D0,.0D0)
      DPSI=CPSI
      CPSS=DPSI*DPSI
      IF(CPSS.GT.1.D0)CPSS=1.D0
      SPSI=DSQRT(1.D0-CPSS)
      IF(DABS(CPSI).LT..999999)GO TO 10
      RHS=RHK*RHK
      RH2=SPSI*(T1+T3)/2.
      RHS=RHS+RH2*RH2
      SGN=1.
      IF(CPSI.GT..0)GO TO 80
      SGN=-1.
      T(1)=-T3
      T(2)=-T2
      T(3)=-T1
      GO TO 80
   10 D=RHK
      DSQ=D*D
      CD=D/SPSI
      BD=CD*DPSI
      EB=DEXP(-BD)
      EC=DEXP(-CD)
      CST=-ETA/(16.*PI*SDK*SDKT)
      TA=T1
      TB=T2
      DO 70 ITT=1,2
      IF(ITT.EQ.1)ET1=DCMPLX(DCOS(TA),DSIN(TA))
      IF(ITT.EQ.2)ET2=DCMPLX(DCOS(TB),DSIN(TB))
      TD1=TA
      TD2=TB
      TS1=TD1*TD1
      TS2=TD2*TD2
      SA=S1
      SB=S2
      DO 60 ISS=1,2
      IF(ISS.EQ.1)ES1=DCMPLX(DCOS(SA),DSIN(SA))
      IF(ISS.EQ.2)ES2=DCMPLX(DCOS(SB),DSIN(SB))
      DO 20 K=1,2
      DO 20 L=1,2
   20 E(K,L)=(.0D0,.0D0)
      SI=SA
      DO 50 I=1,2
      FI=(-1)**I
      SDI=SI
      SIS=SDI*SDI
      ST1=2.*SDI*TD1*DPSI
```

```
      ST2=2.*SDI*TD2*DPSI
      R1=DSQRT(DSQ+SIS+TS1-ST1)
      R2=DSQRT(DSQ+SIS+TS2-ST2)
      EK=EB
      DO 40 K=1,2
      FK=(-1)**K
      SK=FK*SDI
      EL=EC
      DO 30 L=1,2
      FL=(-1)**L
      EKL=EK*EL
      XX=FK*BD+FL*CD
      TL1=FL*TD1
      TL2=FL*TD2
      RR1=R1+SK+TL1
      RR2=R2+SK+TL2
      CALL EXPJ (DCMPLX(XX,RR1),DCMPLX(XX,RR2),EXPA)
      CALL EXPJ (DCMPLX(-XX,RR1),DCMPLX(-XX,RR2),EXPB)
      E(K,L)=E(K,L)+FI*(EXPA*EKL+EXPB/EKL)
30    EL=1./EC
40    EK=1./EB
      IF(I.EQ.ISS)GO TO 50
      ZD=SDI*DPSI
      ZC=ZD
      EGZI=DCMPLX(DCOS(ZC),DSIN(ZC))
      RR1=R1+ZD-TD1
      RR2=R2+ZD-TD2
      CALL EXPJ (DCMPLX(.0D0,RR1),DCMPLX(.0D0,RR2),EXPB)
      RR1=R1-ZD+TD1
      RR2=R2-ZD+TD2
      CALL EXPJ (DCMPLX(.0D0,RR1),DCMPLX(.0D0,RR2),EXPA)
      F(I,1)=(.0D0,2.D0)*SDK*EXPA/EGZI
      F(I,2)=(.0D0,2.D0)*SDK*EXPB*EGZI
50    SI=SB
      IF(ITT.EQ.2)GO TO 54
      IF(ISS.EQ.1)P22=CST*((F(2,1)+E(2,2)*ES1-E(1,2)/ES1)*ET1
     A+(-F(2,2)-E(2,1)*ES1+E(1,1)/ES1)/ET1)
      IF(ISS.EQ.2)P12=CST*((-F(1,1)-E(2,2)*ES2+E(1,2)/ES2)*ET1
     B+( F(1,2)+E(2,1)*ES2-E(1,1)/ES2)/ET1)
      GO TO 58
54    IF(ISS.EQ.1)P21=CST*((-F(2,1)-E(2,2)*ES1+E(1,2)/ES1)*ET2
     C+( F(2,2)+E(2,1)*ES1-E(1,1)/ES1)/ET2)
      IF(ISS.EQ.2)P11=CST*((F(1,1)+E(2,2)*ES2-E(1,2)/ES2)*ET2
     D+(-F(1,2)-E(2,1)*ES2+E(1,1)/ES2)/ET2)
58    SA=S2
60    SB=S3
      TA=T2
70    TB=T3
      Z12=P11+P12+P21+P22
      RETURN
80    DO 100 I=1,3
      SI=S(I)
      CI=1.
      IF(I.EQ.2)CI=-2.*CDK
      CQX=(.0D0,.0D0)
      DO 90 J=1,3
      TJ=T(J)
      CJ=1.
      IF(J.EQ.2)CJ=-2.*CDKT
      DZ=TJ-SI
      R=DSQRT(RHS+DZ*DZ)
      X=R+DZ
      IF(DZ.LT..0)X=RHS/(R-DZ)
      CALL CISI (COSI,CIN,SINI,X)
      EP=DCMPLX(COSI,-SINI)
      X=R-DZ
```

```
      IF(DZ.GT..0)X=RHS/(R+DZ)
      CALL CISI (COSI,CIN,SINI,X)
      EM=DCMPLX(COSI,-SINI)
      EGDZ=DCMPLX(DCOS(DZ),DSIN(DZ))
   90 CQX=CQX+CJ*(EP*EGDZ+EM/EGDZ)
  100 Z12=Z12+CI*CQX
      Z12= SGN*ETA*Z12/(8.*PI*SDK*SDKT)
      RETURN
      END
C
C
```

```
C                                33
C
      SUBROUTINE SKEWS(S1,S3,T1,T3,RHK,CDK,SDK,CDKT,SDKT,CPSI,Z12)      SKEWS.1
C     SKEWS CALCULATES Z12 = MUTUAL IMPEDANCE OF CENTER-FED
C     COPLANAR-SKEW SINUSOIDAL DIPOLES WITH UNEQUAL LENGTHS.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 Z12,EIN,EGDZ,CQX,EJXX,EM,EP
      DIMENSION S(3),T(3)
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      S(1)=S1
      S(2)=(S1+S3)/2.
      S(3)=S3
      T(1)=T1
      T(2)=(T1+T3)/2.
      T(3)=T3
      Z12=(.0D0,.0D0)
      DPSI=CPSI
      IF(DABS(CPSI).LT..999999)GO TO 10
      RHS=RHK*RHK
      CPSS=DPSI*DPSI
      IF(CPSS.GT.1.D0)CPSS=1.D0
      SPSI=DSQRT(1.D0-CPSS)
      RH2=SPSI*(T1+T3)/2.
      RHS=RHS+RH2*RH2
      SGN=1.
      IF(CPSI.GT..0)GO TO 60
      SGN=-1.
      T(1)=-T3
      T(2)=-T(2)
      T(3)=-T1
      GO TO 60
   10 DO 50 I=1,3
      SI=S(I)
      SIS=SI*SI
      CI=1.
      IF(I.EQ.2)CI=-2.*CDK
      DO 50 J=1,3
      TJ=T(J)
      TJS=TJ*TJ
      R=DSQRT(SIS+TJS-2.*SI*TJ*DPSI)
      CJ=1.
      IF(J.EQ.2)CJ=-2.*CDKT
      CQX=(.0D0,.0D0)
      DO 40 K=1,2
      FK=(-1.)**K
      DO 40 L=1,2
      FL=(-1.)**L
      XXD=FK*SI+FL*TJ
      XX=XXD
      EJXX=DCMPLX(DCOS(XX),DSIN(XX))
      XXX=R+XXD
      X=DABS(XXX)
      CALL CISI (COSI,CIN,SINI,X)
      IF(XXX.LT..0)SINI=-SINI
      CQX=CQX+DCMPLX(COSI,-SINI)*EJXX*FK*FL
   40 CONTINUE
      Z12=Z12+CQX*CI*CJ
   50 CONTINUE
      Z12=-ETA*Z12/(8.*PI*SDK*SDKT)
      RETURN
   60 DO 80 I=1,3
      SI=S(I)
      CI=1.
      IF(I.EQ.2)CI=-2.*CDK
      CQX=(.0D0,.0D0)
```

```
      DO 70 J=1,3
      TJ=T(J)
      CJ=1.
      IF(J.EQ.2)CJ=-2.*CDKT
      DZ=TJ-SI
      R=DSQRT(RHS+DZ*DZ)
      X=R+DZ
      IF(DZ.LT..0)X=RHS/(R-DZ)
      CALL CISI (COSI,CIN,SINI,X)
      EP=DCMPLX(COSI,-SINI)
      X=R-DZ
      IF(DZ.GT..0)X=RHS/(R+DZ)
      CALL CISI (COSI,CIN,SINI,X)
      EM=DCMPLX(COSI,-SINI)
      EGDZ=DCMPLX(DCOS(DZ),DSIN(DZ))
   70 CQX=CQX+CJ*(EP*EGDZ+EM/EGDZ)
   80 Z12=Z12+CI*CQX
      Z12=SGN*ETA*Z12/(8.*PI*SDK*SDKT)
      RETURN
      END
C
C
```

```
C
C                                    35
      SUBROUTINE SKEWT(AK,S1,S3,T1,T3,CDK,SDK,CDKD,SDKD,IWZ,Z12)      SKEWT
C     SKEWT CALCULATES Z12 = MUTUAL IMPEDANCE OF WIRE DIPOLE
C     AND DISK DIPOLE.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 P12,Z12,Q12
      DATA PI,P3/3.14159265359,9.42477796077/
  2   FORMAT(1X,8F10.2)
  5   FORMAT(1H0)
  6   FORMAT(5X,'PRINTOUT FROM SKEWT')
  7   FORMAT(5X,'WIRE DIPOLE TO DISK DIPOLE')
      IF(IWZ.LE.0)GO TO 20
      WRITE(17,6)
      WRITE(17,7)
      WRITE(17,5)
 20   RMN=AK/100.
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      Z12=(.0D0,.0D0)
      PH=.0
C  WIRE DIPOLE TO DISK DIPOLE.
      DO 80 IPH=1,2
      Q12=(.0D0,.0D0)
      SGI=-1.
      DO 60 I=1,NPP
      WF=3.+SGI
      IF(I.EQ.1)WF=1.
      IF(I.EQ.NPP)WF=1.
      RH=AK*DSIN(PH)
      IF(I.EQ.1)RH=RH+RMN
      AC=AK*DCOS(PH)
      V1=T1-AC
      V3=T3-AC
      IF(I.EQ.1 .AND. IPH.EQ.2)GO TO 50
      CALL SKEW (S1,S3,V1,V3,RH,CDK,SDK,CDKD,SDKD,.0D0,P12)
 50   Q12=Q12+WF*P12
      PHD=57.29578*PH
      IF(IWZ.GT.0)WRITE(17,2)PHD,P12
      SGI=-SGI
 60   PH=PH+DPH
      Z12=Z12+DPH*Q12/P3
      RMN=.0
      DPH=(PI-PHA)/NPH
 80   PH=PHA
      IF(IWZ.GT.0)WRITE(17,5)
      RETURN
      END
C
C
```

```
C
C
      SUBROUTINE SPART(AK,DKD,DKW,MAX,IWZ,Z,Z1)                    SPART.1
C     SPART CALCULATES Z = MUTUAL IMPEDANCE OF TWO WIRE DIPOLE MODES,
C     AND Z1 = MUTUAL IMPEDANCE BETWEEN A WIRE DIPOLE MODE AND MODE 1.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 EID(20),EM(20),EP(20),Z(1),Z1(1)
      COMPLEX*16 CEM,CEP,EMD,EPD,EMD2,EPD2,Z11,Z22,G11,Q11
      DIMENSION CID(20),SID(20),CM(20),CP(20),SM(20),SP(20)
      DATA GAM,P2/.577215664,1.57079632/
      DATA ETA,PI/376.727,3.14159/
      IDM=20
    1 FORMAT(3X,'MUST INCREASE DIMENSIONS IN SUBROUTINE SPART')
    2 FORMAT(3X,'ACTUAL DIMENSION IDM = ',I5,6X,
     2'REQUIRED DIMENSION MAX2 = ',I5)
    3 FORMAT(1X,8F10.2)
    5 FORMAT(1H0)
    6 FORMAT(5X,'PRINTOUT FROM SPART')
    7 FORMAT(5X,'FIRST:  WIRE DIPOLE TO MODE ONE')
    8 FORMAT(5X,'THEN:  WIRE DIPOLE TO WIRE DIPOLE')
      IF(MAX.LE.0)RETURN
      MAX2=MAX+2
      DO 14 I=1,MAX
      Z1(I)=(.0D0,.0D0)
   14 Z(I)=(.0D0,.0D0)
      IF(MAX2.LE.IDM)GO TO 16
      WRITE(17,1)
      WRITE(17,2)IDM,MAX2
      RETURN
   16 DK=DKW
      IF(IWZ.LE.0)GO TO 18
      WRITE(17,6)
      WRITE(17,7)
      WRITE(17,8)
      WRITE(17,5)
   18 TDK=2.*DK
      SDKD=DSIN(DKD)
      S11=.0
      S13=TDK
      S21=DK
      S23=3.*DK
      DO 20 N=1,MAX2
      I=N-1
      DZ=I*DK
      CID(N)=DCOS(DZ)
      SID(N)=DSIN(DZ)
   20 EID(N)=DCMPLX(CID(N),SID(N))
      CDK=DCOS(DK)
      SDK=DSIN(DK)
      EPD=DCMPLX(CDK,SDK)
      EMD=DCMPLX(CDK,-SDK)
      EPD2=EPD*EPD
      EMD2=EMD*EMD
      CEM=2.*CDK+EMD
      CEP=2.*CDK+EPD
      AK2=AK*AK
      CSS=ETA/(8.*PI*SDK*SDK)
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      PH=.0
      DO 100 JPH=1,2
      CST=DPH*ETA/(24.*PI*PI*SDK*SDK)
```

```
      C22=DPH/(3.*PI)
      SGN=-1.
      DO 80 IPH=1,NPP
      CPH=DCOS(PH)
      SPH=DSIN(PH)
      IF(IPH.GT.1)GO TO 30
      IF(JPH.GT.1)GO TO 30
      PH0=DPH/10.
      CPH=DCOS(PH0)
      SPH=DSIN(PH0)
30    T1=AK*(1.-CPH)
      T2=T1+DKD
      RH=AK*SPH
      DRG=2.*AK2*(1.-CPH)
      RK=DSQRT(DRG)
      RS=DRG
      WF=3.+SGN
      IF(IPH.EQ.1)WF=1.
      IF(IPH.EQ.NPP)WF=1.
      WST=WF*CST
      W22=WF*C22
      DO 40 N=1,MAX2
      I=N-1
      DZ=I*DK
      DZS=DZ*DZ
      R=DSQRT(RS+DZS)
      ARG=R+DZ
      IF(N.EQ.1)ARG=RK
      CALL CISI(CP(N),CIN,SP(N),ARG)
      EP(N)=DCMPLX(CP(N),-SP(N))
      IF(N.GT.1)GO TO 38
      CM(1)=CP(1)
      SM(1)=SP(1)
      EM(1)=EP(1)
      GO TO 40
38    ARG=RS/ARG
      CALL CISI(CM(N),CIN,SM(N),ARG)
      EM(N)=DCMPLX(CM(N),-SM(N))
40    CONTINUE
      R=4.*(-CM(2)+2.*CP(1)-CP(2))
     A+2.*CID(3)*(+CM(3)-2.*CM(2)+2.*CP(1)-2.*CP(2)+CP(3))
     B+2.*SID(3)*(-SM(3)+2.*SM(2)-2.*SP(2)+SP(3))
      X=4.*(SM(2)-2.*SP(1)+SP(2))
     C+2.*CID(3)*(-SM(3)+2.*SM(2)-2.*SP(1)+2.*SP(2)-SP(3))
     D+2.*SID(3)*(-CM(3)+2.*CM(2)-2.*CP(2)+CP(3))
      Q11=CSS*DCMPLX(R,X)
      Z(1)=Z(1)+WST*DCMPLX(R,X)
      Z11=-6.*CDK*EP(1)+2.*(EPD+CDK)*EP(2)-EPD*EP(3)+
     22.*(EMD+CDK)*EM(2)-EMD*EM(3)
      CALL ZSDM(S11,S13,T1,T2,RH,CDK,SDK,SDKD,.0D0,Z22)
      G11=CSS*Z11-Z22
      Z1(1)=Z1(1)+WST*Z11-W22*Z22
      PHD=57.29578*PH
      IF(IWZ.GT.0)WRITE(17,3)PHD,G11,Q11
      IF(MAX.EQ.1)GO TO 70
      R=2.*CID(2)*(-CM(3)+3.*CM(2)-4.*CP(1)+3.*CP(2)-CP(3))
     E+2.*SID(2)*(+SM(3)-2.*SM(2)+2.*SP(2)-SP(3))
     F+CID(4)*(+CM(4)-2.*CM(3)+CM(2)+CP(2)-2.*CP(3)+CP(4))
     G+SID(4)*(-SM(4)+2.*SM(3)-SM(2)+SP(2)-2.*SP(3)+SP(4))
      X=2.*CID(2)*(SM(3)-3.*SM(2)+4.*SP(1)-3.*SP(2)+SP(3))
     H +2.*SID(2)*(CM(3)-2.*CM(2)+2.*CP(2)-CP(3))
     I+CID(4)*(-SM(4)+2.*SM(3)-SM(2)-SP(2)+2.*SP(3)-SP(4))
     J+SID(4)*(-CM(4)+2.*CM(3)-CM(2)+CP(2)-2.*CP(3)+CP(4))
      Z(2)=Z(2)+WST*DCMPLX(R,X)
      Z11=2.*EP(1)+CEM*(EMD*EM(3)-EPD*EP(2))+
     2CEP*(EPD*EP(3)-EMD*EM(2))-EPD2*EP(4)-EMD2*EM(4)
```

```
      CALL ZSDM(S21,S23,T1,T2,RH,CDK,SDK,SDKD,.0D0,Z22)
      Z1(2)=Z1(2)+WST*Z11-W22*Z22
      IF(MAX.EQ.2)GO TO 70
      S1=DK
      DO 60 N=3,MAX
      M1=N-1
      M2=N-2
      N1=N+1
      N2=N+2
      Z11=EP(M1)*EID(M1)+EM(M1)/EID(M1)+CEP*(EP(N1)*EID(N)-EM(N)/EID(N))
     2-CEM*(EP(N)*EID(N)-EM(N1)/EID(N))-EP(N2)*EID(N1)-EM(N2)/EID(N1)
      S1=S1+DK
      S3=S1+TDK
      CALL ZSDM(S1,S3,T1,T2,RH,CDK,SDK,SDKD,.0D0,Z22)
      Z1(N)=Z1(N)+WST*Z11-W22*Z22
      CPA=CP(M2)-2.*CP(M1)+CP(N)
      CPB=2.*CP(N)-CP(M1)-CP(N1)
      CPC=CP(N2)-2.*CP(N1)+CP(N)
      CMA=CM(M2)-2.*CM(M1)+CM(N)
      CMB=2.*CM(N)-CM(N1)-CM(M1)
      CMC=CM(N2)-2.*CM(N1)+CM(N)
      SPA=SP(M2)-2.*SP(M1)+SP(N)
      SPB=2.*SP(N)-SP(M1)-SP(N1)
      SPC=SP(N2)-2.*SP(N1)+SP(N)
      SMA=SM(M2)-2.*SM(M1)+SM(N)
      SMB=2.*SM(N)-SM(N1)-SM(M1)
      SMC=SM(N2)-2.*SM(N1)+SM(N)
      R=CID(M2)*(CPA+CMA)+2.*CID(N)*(CPB+CMB)+2.*SID(N)*(SPB-SMB)
     K +CID(N2)*(CPC+CMC)+SID(N2)*(SPC-SMC)
      IF(N.GT.3)R=R+SID(M2)*(SPA-SMA)
      X=-CID(M2)*(SPA+SMA)-2.*CID(N)*(SPB+SMB)+2.*SID(N)*(CPB-CMB)
     L -CID(N2)*(SPC+SMC)+SID(N2)*(CPC-CMC)
      IF(N.GT.3)X=X+SID(M2)*(CPA-CMA)
   60 Z(N)=Z(N)+WST*DCMPLX(R,X)
   70 PH=PH+DPH
   80 SGN=-SGN
      DPH=(PI-PHA)/NPH
  100 PH=PHA
      IF(IWZ.GT.0)WRITE(17,5)
      RETURN
      END
C
C
```

```
C                                        39
C
      SUBROUTINE ZSDM(S1,S3,T1,T2,RHK,CDK,SDK,SDKT,CPSI,Z12)            ZSDM.1
C     CALCULATES Z12 = MUTUAL IMPEDANCE BETWEEN SINUSOIDAL DIPOLE
C     AND SINUSOIDAL MONOPOLE WITH SKEW ORIENTATION.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 E(2,2),F(2,2),ES2,ET2,EXPA,EXPB,EGZI,ES1
      COMPLEX*16 CQX,EJXX,Z12,EP1,EM1,P11,P21,EGDZ,EM,EP
      DIMENSION S(3)
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      S(1)=S1
      S2=(S1+S3)/2.
      S(2)=S2
      S(3)=S3
      Z12=(.0D0,.0D0)
      DPSI=CPSI
      CPSS=DPSI*DPSI
      IF(CPSS.GT.1.D0)CPSS=1.D0
      SPSI=DSQRT(1.D0-CPSS)
      IF(DABS(CPSI).LT..999999)GO TO 10
      RHS=RHK*RHK
      RH2=SPSI*(T1+T2)/2.
      RHS=RHS+RH2*RH2
      SGN=1.
      IF(CPSI.GT..0)GO TO 80
      S(1)=-S3
      S(2)=-S2
      S(3)=-S1
      SGN=-1.
      GO TO 80
   10 D=RHK
      DSQ=D*D
      CD=D/SPSI
      BD=CD*DPSI
      EB=DEXP(-BD)
      EC=DEXP(-CD)
      TD1=T1
      TD2=T2
      TS1=TD1*TD1
      TS2=TD2*TD2
      CST=-ETA/(16.*PI*SDK*SDKT)
      SA=S1
      SB=S2
      ET2=DCMPLX(DCOS(T2),DSIN(T2))
      DO 60 ISS=1,2
      IF(ISS.EQ.1)ES1=DCMPLX(DCOS(SA),DSIN(SA))
      IF(ISS.EQ.2)ES2=DCMPLX(DCOS(SB),DSIN(SB))
      DO 20 K=1,2
      DO 20 L=1,2
   20 E(K,L)=(.0D0,.0D0)
      SI=SA
      DO 50 I=1,2
      FI=(-1)**I
      SDI=SI
      SIS=SDI*SDI
      ST1=2.*SDI*TD1*DPSI
      ST2=2.*SDI*TD2*DPSI
      R1=DSQRT(DSQ+SIS+TS1-ST1)
      R2=DSQRT(DSQ+SIS+TS2-ST2)
      EK=EB
      DO 40 K=1,2
      FK=(-1)**K
      SK=FK*SDI
      EL=EC
      DO 30 L=1,2
```

```
      FL=(-1)**L
      EKL=EK*EL
      XX=FK*BD+FL*CD
      TL1=FL*TD1
      TL2=FL*TD2
      RR1=R1+SK+TL1
      RR2=R2+SK+TL2
      CALL EXPJ(DCMPLX(XX,RR1),DCMPLX(XX,RR2),EXPA)
      CALL EXPJ(DCMPLX(-XX,RR1),DCMPLX(-XX,RR2),EXPB)
      E(K,L)=E(K,L)+FI*(EXPA*EKL+EXPB/EKL)
   30 EL=1./EC
   40 EK=1./EB
      IF(I.EQ.ISS)GO TO 50
      ZD=SDI*DPSI
      ZC=ZD
      EGZI=DCMPLX(DCOS(ZC),DSIN(ZC))
      RR1=R1+ZD-TD1
      RR2=R2+ZD-TD2
      CALL EXPJ(DCMPLX(.0D0,RR1),DCMPLX(.0D0,RR2),EXPB)
      RR1=R1-ZD+TD1
      RR2=R2-ZD+TD2
      CALL EXPJ(DCMPLX(.0D0,RR1),DCMPLX(.0D0,RR2),EXPA)
      F(I,1)=(.0D0,2.D0)*SDK*EXPA/EGZI
      F(I,2)=(.0D0,2.D0)*SDK*EXPB*EGZI
   50 SI=SB
      IF(ISS.EQ.1)
     AP21=CST*((-F(2,1)-E(2,2)*ES1+E(1,2)/ES1)*ET2
     B+( F(2,2)+E(2,1)*ES1-E(1,1)/ES1)/ET2)
      IF(ISS.EQ.2)
     CP11=CST*(( F(1,1)+E(2,2)*ES2-E(1,2)/ES2)*ET2
     D+(-F(1,2)-E(2,1)*ES2+E(1,1)/ES2)/ET2)
      SA=S2
   60 SB=S3
      Z12=P11+P21
      RETURN
   80 DO 100 I=1,3
      CI=1.
      IF(I.EQ.2)CI=-2.*CDK
      SI=S(I)
      TJ=T1
      DO 90 J=1,2
      DZ=TJ-SI
      R=DSQRT(RHS+DZ*DZ)
      X=R+DZ
      IF(DZ.LT..0)X=RHS/(R-DZ)
      CALL CISI(COSI,CIN,SINI,X)
      EP=DCMPLX(COSI,-SINI)
      X=R-DZ
      IF(DZ.GT..0)X=RHS/(R+DZ)
      CALL CISI(COSI,CIN,SINI,X)
      EM=DCMPLX(COSI,-SINI)
      IF(J.EQ.2)GO TO 90
      EP1=EP
      EM1=EM
   90 TJ=T2
      X=T2-SI
      EGDZ=DCMPLX(DCOS(X),DSIN(X))
      Z12=Z12+CI*((EP-EP1)*EGDZ+(EM-EM1)/EGDZ)
  100 CONTINUE
      Z12= SGN*ETA*Z12/(8.*PI*SDK*SDKT)
      RETURN
      END
C
C
```

```
C                                      41
C
      SUBROUTINE ZSMM(S1,S2,T1,T2,D,CDS,SDS,SDT,CPSI,P11)           ZSMM.1
C     CALCULATES MUTUAL IMPEDANCE OF COPLANAR-SKEW
C     SINUSOIDAL MONOPOLES.
        IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 E(2,2),F(2,2),GAM,P11,P12,P21,P22
      COMPLEX*16 EGZI,ES1,ES2,ET1,ET2,EXPA,EXPB
      COMPLEX*16 EGDZ,EM,EP,EM1,EP1
      DATA ETA,GAM,PI/376.730366239,(.0D0,1.D0),3.14159265359/
      DD=D
      DPQ=DD*DD
      DPSI=CPSI
      CPSS=DPSI*DPSI
      IF(CPSS.GT.1.D0)CPSS=1.D0
      SPSI=DSQRT(1.D0-CPSS)
      SGDS=SDS
      IF(S2.LT.S1)SGDS=-SDS
      SGDT=SDT
      IF(T2.LT.T1)SGDT=-SDT
      IF(DABS(CPSI).LT..999999)GO TO 6
      D0=SPSI*(T1+T2)/2.
      DSQ=DPQ+D0*D0
      GO TO 110
    6 ES1=DCMPLX(DCOS(S1),DSIN(S1))
      ES2=DCMPLX(DCOS(S2),DSIN(S2))
      ET1=DCMPLX(DCOS(T1),DSIN(T1))
      ET2=DCMPLX(DCOS(T2),DSIN(T2))
      ID=1
      IF(D.EQ..0)ID=0
      TD1=T1
      TD2=T2
      CD=DD/SPSI
      C=CD
      BD=CD*DPSI
      B=BD
      EB=.0
      EC=.0
      IF(ID.EQ.0)GO TO 8
      EB=DEXP(-B)
      EC=DEXP(-C)
    8 DO 10 K=1,2
      DO 10 L=1,2
   10 E(K,L)=(.0D0,.0D0)
      TS1=TD1*TD1
      TS2=TD2*TD2
      SI=S1
      DO 100 I=1,2
      FI=(-1)**I
      SDI=SI
      SIS=SDI*SDI
      ST1=2.*SDI*TD1*DPSI
      ST2=2.*SDI*TD2*DPSI
      R1=DSQRT(DPQ+SIS+TS1-ST1)
      R2=DSQRT(DPQ+SIS+TS2-ST2)
      EK=EB
      DO 50 K=1,2
      FK=(-1)**K
      SK=FK*SDI
      EL=EC
      DO 40 L=1,2
      FL=(-1)**L
      EKL=EK*EL
      XX=FK*BD+FL*CD
      TL1=FL*TD1
```

```
      TL2=FL*TD2
      RR1=R1+SK+TL1
      RR2=R2+SK+TL2
      AXX=DABS(XX)
      IF(AXX.GT.DABS(RR1)/100.)GO TO 28
      IF(AXX.GT.DABS(RR2)/100.)GO TO 28
      IF(AXX.GT..001)GO TO 28
      IF(RR1/RR2.LT..0)GO TO 28
      CALL CISI(COS1,CIN,SIN1,RR1)
      CALL CISI(COS2,CIN,SIN2,RR2)
      EXPA=DCMPLX(COS2-COS1,SIN1-SIN2)
      E(K,L)=E(K,L)+FI*EXPA*(EKL+1./EKL)
      GO TO 40
28    CALL EXPJ(DCMPLX(XX,RR1),DCMPLX(XX,RR2),EXPA)
      CALL EXPJ(DCMPLX(-XX,RR1),DCMPLX(-XX,RR2),EXPB)
      E(K,L)=E(K,L)+FI*(EXPA*EKL+EXPB/EKL)
40    EL=1./EC
50    EK=1./EB
      IF(I.EQ.2)GO TO 100
      ZD=SDI*DPSI
      ZC=ZD
      EGZI=DCMPLX(DCOS(ZC),DSIN(ZC))
      RR1=R1+ZD-TD1
      RR2=R2+ZD-TD2
      CALL CISI(COS1,CIN,SIN1,RR1)
      CALL CISI(COS2,CIN,SIN2,RR2)
      EXPB=DCMPLX(COS2-COS1,SIN1-SIN2)
      RR1=R1-ZD+TD1
      RR2=R2-ZD+TD2
      CALL CISI(COS1,CIN,SIN1,RR1)
      CALL CISI(COS2,CIN,SIN2,RR2)
      EXPA=DCMPLX(COS2-COS1,SIN1-SIN2)
      F(I,1)=2.*SGDS*(.0D0,1.D0)*EXPA/EGZI
      F(I,2)=2.*SGDS*(.0D0,1.D0)*EXPB*EGZI
100   SI=S2
      CST=-ETA/(16.*PI*SGDS*SGDT)
      P11=CST*(( F(1,1)+E(2,2)*ES2-E(1,2)/ES2)*ET2
     A+(-F(1,2)-E(2,1)*ES2+E(1,1)/ES2)/ET2)
      RETURN
110   IF(CPSI.LT.0.)GO TO 120
      TA=T1
      TB=T2
      GO TO 130
120   TA=-T1
      TB=-T2
      SGDT=-SGDT
130   SI=S1
      CI=-CDS
      P11=(.0D0,.0D0)
      DO 150 I=1,2
      TJ=TA
      DO 140 J=1,2
      DZ=TJ-SI
      R=DSQRT(DSQ+DZ*DZ)
      X=R+DZ
      IF(DZ.LT..0)X=DSQ/(R-DZ)
      CALL CISI(COSI,CIN,SINI,X)
      EP=DCMPLX(COSI,-SINI)
      X=R-DZ
      IF(DZ.GT..0)X=DSQ/(R+DZ)
      CALL CISI(COSI,CIN,SINI,X)
      EM=DCMPLX(COSI,-SINI)
      IF(J.EQ.2)GO TO 140
      EP1=EP
      EM1=EM
140   TJ=TB
```

```
      X=TB-SI
      EGDZ=DCMPLX(DCOS(X),DSIN(X))
      P11=P11+CI*((EP-EP1)*EGDZ+(EM-EM1)/EGDZ)
      CI=1.
  150 SI=S2
      P11=ETA*P11/(8.*PI*SGDS*SGDT)
      RETURN
      END
C
C
```

APPENDIX B

COMPUTER PROGRAM RICHMOND4 FOR THE FAR-ZONE FIELD OF A MONOPOLE ELEMENT ON A DISK GROUND PLANE ABOVE FLAT EARTH

COMPUTER PROGRAM RICHMOND4: MONOPOLE ANTENNA ON CIRCULAR DISK OVER FLAT EARTH

(IMPEDANCE, GAIN, AND FAR-FIELD PATTERNS)

by
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February 16, 1990
Revised September 21, 1990

INTRODUCTION [1]

Appendix I presents the computer program RICHMOND4. This FORTRAN program calculates the current distribution, terminal impedance, and directive gain $G(\theta)$ of a base-fed monopole antenna mounted at the center of a circular disk over the flat lossy earth. The detailed theory behind this moment-method solution is presented in the following paper: (J. H. Richmond, "Monopole Antenna on Circular Disk over Flat Earth," IEEE Transactions, Vol. AP-33, pp. 633-637, June 1985.)

To assist the user, comment statements have been inserted in the main program and the subroutines. Only a few additional brief comments will be required in this Introduction.

RICHMOND4 performs all calculations with double precision. In this program the notation corresponds closely with the notation in the above paper, with one exception: In the paper z_o denotes the height of the circular disk above the surface of the earth, whereas in the program HDL denotes z_o/λ. (The wavelength in free space is denoted by λ or WAVM.)

[1] Appreciation is expressed to The MITRE Corporation for sponsoring this report.

RICHMOND4 requires all the subroutines used by RICHMOND3. In addition, RICHMOND4 requires the following additional subroutines which are listed after the main program in Appendix I: EDISK1, EDISK2, GAIN1, and GAIN2.

Subroutines EDISK1 and GAIN1 calculate the far-zone electric field intensity $E_\theta(\theta)$ for the monopole on the circular disk in free space, with $0 \leq \theta \leq \pi$. In these calculations, the factor $\exp(-jkr)/(kr)$ is suppressed. For the monopole on a circular disk over the flat earth, EDISK2 and GAIN2 calculate $E_\theta(\theta)$ with $0 \leq \theta \leq \pi/2$. In GAIN1 and GAIN2, ET denotes the quantity $kr \ \exp(jkr) \ E_\theta(r, \theta)$, which may be called "the normalized far-zone electric field intensity" corresponding to the space wave.

GAIN1 and GAIN2 each makes two passes through the range of angles θ. On the first pass ($M = 1$), the time-average radiated power P_r is calculated via numerical integration using appropriately small increments DTH in the angle θ. On the second pass ($M = 2$), the directive gain $D(\theta)$ is calculated and stored using the angular increments DTHD specified in the input data.

Tables I and II show numerical results (with RICHMOND4) for circular disks with radii $ka = 1.5$ and 3.0 respectively, where k denotes the wavenumber in free space. For the monopole on a disk in free space, the radiation resistance R(RAD) and the directive gain GAIN agree closely with Tables A2-6 and A2-12 in the following:
(M. M. Weiner et al., "Monopole Elements on Circular Ground Planes," Artech House, 1987.)

In addition, Tables I and II list the radiation resistance and gain for the monopole on a disk on the flat lossy earth, as well as the antenna terminal impedance in free space and on the flat earth. Let us define the radiation efficiency to be the ratio between the power radiated (via the "space wave" into the free-space region) and the power input at the antenna terminals. This radiation efficiency, then, is equal to the ratio of the radiation resistance and the real part of the antenna terminal impedance. In Tables I and II the radiation efficiency is 100% for the antenna in free space. For

the antenna on flat earth the radiation efficiency is 25.1% with the smaller disk, and 46.6% with the larger disk.

RICHMOND4 has been tested with several of the examples in the published paper cited earlier (Richmond, IEEE, 1985), with excellent agreement on the antenna terminal impedance. In addition, the RICHMOND4 calculations converge properly as the number of segments (NSD and NSW) increases.

In the calculation of the normalized far-zone electric field intensity ET, GAIN1 and GAIN2 include only the "space wave" component of the field. The effects of the round earth and the ionosphere are not included. Even with flat earth, as the observer approaches the air-earth interface, the "ground wave" or "surface wave" field may become significant, but is not included in ET.

TABLE I. Numerical Results with ka = 1.5

DOUBLE PRECISION
MONOPOLE ON CIRCULAR DISK

NSD	NSW	AL	CK	CL	HL	HDL
8	2	1.E-6	1.5	0.2387	0.25	0.0

R(RAD)	R	X	
22.0733	22.0710	13.7758	(IN FREE SPACE)

(IN FREE SPACE)

THETA	GAIN
0.0000	0.0000
10.0000	0.0582
20.0000	0.2202
30.0000	0.4521
40.0000	0.7103
50.0000	0.9540
60.0000	1.1543
70.0000	1.2967
80.0000	1.3781
90.0000	1.4008
100.0000	1.3674
110.0000	1.2775
120.0000	1.1306
130.0000	0.9303
140.0000	0.6906
150.0000	0.4387
160.0000	0.2135
170.0000	0.0564
180.0000	0.0000

(ON FLAT EARTH)

THETA	GAIN
0.0000	0.0000
10.0000	0.1550
20.0000	0.6084
30.0000	1.3146
40.0000	2.1628
50.0000	2.9360
60.0000	3.2924
70.0000	2.8331
80.0000	1.3814
90.0000	0.0000

NSD	NSW	AL	BAR	CL	ER	FMC	HDL	HL	SIG
8	2	1.E-6	3.0	0.2387	4.	10.	0.0	0.25	.001

R(RAD)	R	X	
7.4736	29.7531	12.3685	(ON FLAT EARTH)

TABLE II. Numerical Results with ka = 3

DOUBLE PRECISION

MONOPOLE ON CIRCULAR DISK

NSD	NSW	AL	CX	CL	HL	HDL
8	2	1.E-6	3.	0.4775	0.2500	0.0

R (RAD)	R	X	
40.0556	40.0120	33.0500	(IN FREE SPACE)

(IN FREE SPACE)

THETA	GAIN
0.0000	0.0000
10.0000	0.2238
20.0000	0.7637
30.0000	1.3305
40.0000	1.6823
50.0000	1.7417
60.0000	1.5725
70.0000	1.2923
80.0000	1.0069
90.0000	0.7868
100.0000	0.6695
110.0000	0.6649
120.0000	0.7499
130.0000	0.8597
140.0000	0.8925
150.0000	0.7578
160.0000	0.4593
170.0000	0.1391
180.0000	0.0000

(ON FLAT EARTH)

THETA	GAIN
0.0000	0.0000
10.0000	0.2630
20.0000	0.9524
30.0000	1.8271
40.0000	2.6187
50.0000	3.1019
60.0000	3.0919
70.0000	2.4352
80.0000	1.1249
90.0000	0.0000

NSD	NSW	AL	BAR	CL	ER	FMC	HDL	HL	SIG
8	2	1.E-6	3.0	.4775	4.	10.	0.0	.25	.001

R (RAD)	R	X	
18.2413	39.1551	27.5684	(ON FLAT EARTH)

APPENDIX I. RICHMOND4 and the Subroutines

```
C       RICHMOND4
C       MONOPOLE AT CENTER OF CIRCULAR DISK OVER FLAT EARTH.        RICHMOND4.1
C       DOUBLE PRECISION.
C       CURRENT DISTRIBUTION, IMPEDANCE, AND FAR-FIELD PATTERN.
C       LINK: BES10,CISI,CMINV,DZ11,DZDD,DZWD,DZWW,EDISK1,EDISK2,EXPJ,
C       GAIN1,GAIN2,GRILL,QDD,QDM,QMM,SKEW,SKEWS,SKEWT,SFART,ZSDM,ZSMM
        IMPLICIT REAL*8 (A-H), (P-Z)
        COMPLEX*16 CJ(30),VJ(30),ZJ(30),VIJ(30,30),ZIJ(30,30)
        COMPLEX*16 Y11,DET,EC,D11,D12,D21,D22,DZ1J,DV1,W11
        COMPLEX*16 P11,P12,P21,P22,ZDD,ZDM,ZMD,ZMM,Z11,ZD,Z22,Z12,Z21
        DIMENSION FB(500),G(182),LLL(30),MMM(30)
        DATA E0,U0/8.854185233677D-12,1.25663706144D-6/
        DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
        DATA ICC,IFB/30,500/
  1     FORMAT(1X,2I5,8F10.4)
  2     FORMAT(1X,8F10.4)
  5     FORMAT(1H0)
C       AL = RADIUS OF WIRE IN WAVELENGTHS.
C       BAR = RADIUS RATIO FOR COAXIAL FEED CABLE.
C       CL = RADIUS OF CIRCULAR DISK IN WAVELENGTHS = EPSLN/TP.
C       DTHD = INCREMENT IN FAR-FIELD ANGLE THETA (DEGREES).
C       ER = RELATIVE PERMITTIVITY OF EARTH.
C       FMC = FREQUENCY IN MEGAHERTZ.
C       HL = LENGTH OF MONOPOLE IN WAVELENGTHS.
C       HDL = HEIGHT OF DISK ABOVE THE EARTH IN WAVELENGTHS.
C       NSD = NUMBER OF SEGMENTS ON THE DISK.
C       NSW = NUMBER OF SEGMENTS ON THE WIRE.
C       SIG = CONDUCTIVITY OF EARTH, MHO/M.
C       SET DTHD = NEGATIVE TO SKIP THE GAIN CALCULATIONS.
C       SET HDL = NEGATIVE FOR MONOPOLE-DISK IN FREE SPACE,
C        OR HDL = POS. FOR FREE SPACE + FLAT EARTH.
C       SET IWCJ = 1 TO WRITE OUT THE CURRENTS CJ(N),
C        OR IWCJ = 0 TO SUPPRESS WRITEOUT.
C        TL = 1.D-5 FOR EPSLN GREATER THAN OR EQUAL 0.25,
C           = AL.D-4 FOR EPSLN LESS THAN 0.25.

        AL=1.D-6
        BAR=3.
        CL=3./TP
        DTHD=10.
        ER=4.
        FMC=10.
        HL=.25
        HDL=1.D-5
        IWCJ=0
        NSD=8
        NSW=2
        SIG=.001
        TL=1.D-5
        WAVM=300./FMC
        IWZ=0
        NPH=6
        NEQ=NSD+NSW-1
        IF(NEQ.GT.ICC)GO TO 400
        AK=TP*AL
        CK=TP*CL
        HK=TP*HL
        HDK=TP*HDL
        TK=TP*TL
        OMEG=TP*FMC*1.D6
        EC=DCMPLX(ER,-SIG/(OMEG*E0))
        DKD=(CK-AK)/NSD
        DKW=HK/NSW
        RH2=AK+DKD
        IF(RH2.LT.BAR*AK)GO TO 400
        TDKD=2.*DKD
```

```
      CDKD=DCOS(DKD)
      SDKD=DSIN(DKD)
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      MAX=NSW-1
      NA=NSD+1
      CALL QMM(AK,DKD,DKW,CDKD,SDKD,CDK,SDK,TK,IWZ,NPH,Z11)
      ZIJ(1,1)=Z11
      IF(NSD.LE.1)GO TO 100
      S1=AK
      DO 60 J=2,NSD
      S2=S1+DKD
      S3=S1+TDKD
      T1=AK
      DO 50 I=2,J
      T2=T1+DKD
      T3=T1+TDKD
      CALL QDD(CDKD,SDKD,S1,S3,T1,T3,TK,IWZ,NPH,Z22)
      ZIJ(I,J)=Z22
   50 T1=T1+DKD
      CALL QDM(AK,DKD,DKW,CDKD,SDKD,SDK,S1,S3,TK,IWZ,NPH,Z12)
      ZIJ(1,J)=Z12
   60 S1=S1+DKD
  100 IF(NSW.LE.1)GO TO 200
      CALL SPART(AK,DKD,DKW,MAX,IWZ,ZJ,CJ)
      L=0
      DO 160 I=NA,NEQ
      DO 150 J=I,NEQ
      K=J-I+1
  150 ZIJ(I,J)=ZJ(K)
      L=L+1
      ZIJ(1,I)=CJ(L)
  160 CONTINUE
  178 IF(NSD.LE.1)GO TO 200
      Z2=.0
      DO 190 J=NA,NEQ
      Z2=Z2+DKW
      S1=Z2-DKW
      S3=Z2+DKW
      RH2=AK
      DO 180 I=2,NSD
      RH2=RH2+DKD
      T1=RH2-DKD
      T3=RH2+DKD
      CALL SKEWT(AK,S1,S3,T1,T3,CDK,SDK,CDKD,SDKD,IWZ,Z12)
  180 ZIJ(I,J)=Z12
  190 CONTINUE
  200 CALL GRILL(AK,BAR,DKD,DKW,NEQ,NSD,NSW,TK,VJ)
      DO 210 I=1,NEQ
      DO 210 J=I,NEQ
C       WRITE(17,1)I,J,ZIJ(I,J)
      ZIJ(J,I)=ZIJ(I,J)
  210 VIJ(I,J)=ZIJ(I,J)
C       WRITE(6,1)NSD,NSW,AL,CK,CL,HL,HDL
C       WRITE(17,1)NSD,NSW,AL,CK,CL,HL,HDL
C       WRITE(6,5)
C       WRITE(17,5)
      CALL CMINV(CJ,VJ,ZIJ,ICC,IWCJ,1,LLL,MMM,NEQ,DET)
      Y11=CJ(1)
      Z11=1./Y11
C
C     CALCULATE DIRECTIVE GAIN G(N) IN FREE SPACE.
C
      RR1=.0
C       IF(DTHD.LE..0)GO TO 212
C       GAINA = DIRECTIVITY IN FREE SPACE.
```

```
C        THA = ANGLE OF MAXIMUM GAIN IN FREE SPACE.                    MAIN.3
         CALL GAIN1(AK,CK,CJ,DTHD,G,GAINA,HK,NSD,
     2   NSW,NTH,PR,THA,WAVM)
       AJ=CDABS(CJ(1))
       RR1=PR/(AJ*AJ)
  212    CONTINUE
C        WRITE(6,2)HDL,GAINA,RR1,Z11
C        WRITE(15,2)HDL,GAINA,RR1,Z11
C       WRITE(6,5)
C       WRITE(17,5)
       IF(DTHD.LE..0)GO TO 222
       DO 220 N=1,NTH
       TH=DTHD*(N-1)
       WRITE(6,2)TH,G(N)
       WRITE(17,2)TH,G(N)
  220 CONTINUE
       WRITE(6,5)
       WRITE(17,5)
  222 CONTINUE
C
C      CALCULATE DIRECTIVE GAIN G(N) OVER FLAT EARTH.
C
       IF(HDL.LT..0)GO TO 350
         DO 400 NHD=1,4
         HDL=NHD
         HDK=TP*HDL
C DELETE STATEMENT 230 UNLESS THE CURRENT DISTRIBUTION IS TO BE
C   APPROXIMATED BY THE CUR. DIST. FOR ANTENNA IN FREE SPACE.
C  230  IF(NHD.GT.0)GO TO 316
       CALL DZ11(AK,BAR,DKD,DKW,EC,FB,HDK,TK,IFB,D11,DV1)
       ZIJ(1,1)=D11
       IF(NSD.LE.1)GO TO 265
       S2=AK+DKD
       DO 260 J=2,NSD
       T2=AK+DKD
       I12=1
       DO 250 I=2,J
       CALL DZDD(AK,DBET,DKD,DKW,EC,FB,HDK,S2,T2,TK
     2,IFB,I12,KMX,D12,D22)
       IF(I.EQ.2)P12=D12
       ZIJ(I,J)=D22
       I12=2
  250 T2=T2+DKD
       ZIJ(1,J)=P12
  260 S2=S2+DKD
  265 IF(NSW.LE.1)GO TO 278
       DO 276 K=1,MAX
       CALL DZWW(AK,DKD,DKW,EC,HDK,K,TK,ZJ,DZ1J)
       J=NA+K-1
       ZIJ(1,J)=DZ1J
       L=1
       DO 270 I=NA,J
       ZIJ(I,J)=ZJ(L)
  270 L=L+1
  276 CONTINUE
  278 IF(NSD.LE.1)GO TO 300
       Z2=.0
       DO 290 J=NA,NEQ
       Z2=Z2+DKW
       CALL DZWD(AK,DKD,DKW,EC,HDK,NSD,TK,Z2,ZJ)
       DO 280 I=2,NSD
  280 ZIJ(I,J)=ZJ(I)
  290 CONTINUE
  300 DO 310 I=1,NEQ
       DO 308 J=I,NEQ
       Z12=VIJ(I,J)
```

```
      D12=ZIJ(I,J)
C      WRITE(17,1)I,J,Z12,D12
      ZIJ(I,J)=Z12+D12
  308 CONTINUE
  310 CONTINUE
C      WRITE(17,5)
      CALL GRILL(AK,BAR,DKD,DKW,NEQ,NSD,NSW,TK,VJ)
      VJ(1)=VJ(1)+DV1
      DO 315 I=1,NEQ
      DO 312 J=I,NEQ
  312 ZIJ(J,I)=ZIJ(I,J)
  315 CONTINUE
      CALL CMINV(CJ,VJ,ZIJ,ICC,IWCJ,1,LLL,MMM,NEQ,DET)
  316   RR2=.0
C       IF(DTHD.LE..0)GO TO 322
C       GAINB = DIRECTIVITY OVER FLAT EARTH.
C       THB = ANGLE OF MAXIMUM GAIN OVER FLAT EARTH.
        CALL GAIN2(AK,CK,CJ,DTHD,EC,G,GAINB,HDK,HK,
     2  NSD,NSW,NTH,PR,THB,WAVM)
      AJ=CDABS(CJ(1))
      RR2=PR/(AJ*AJ)
        IF(DTHD.LE.0.)GO TO 322
      DO 320 N=1,NTH
      TH=DTHD*(N-1)
      WRITE(6,2)TH,G(N)
      WRITE(17,2)TH,G(N)
  320 CONTINUE
      WRITE(6,5)
      WRITE(17,5)
  322 Y11=CJ(1)
      W11=1./Y11
        DBB=10.*ALOG10(GAINB)
      WRITE(6,2)HDL,DBB,THB,RR2,W11
      WRITE(16,2)HDL,DBB,THB,RR2,W11
C      WRITE(17,1)NSD,NSW,AL,BAR,CL,ER,FMC,HDL,HL,SIG
C      WRITE(17,5)
  350 CONTINUE
  400 CONTINUE
        WRITE(6,5)
        WRITE(16,5)
        DBA=10.*ALOG10(GAINA)
        WRITE(6,2)DBA,THA,RR1,Z11
        WRITE(16,2)DBA,THA,RR1,Z11
  500 CALL EXIT
      END
C
C
```

```
C
      SUBROUTINE GAIN1(AK,CK,CJ,DTHD,G,GAINA,HK,NSD,NSW,          GAIN1.1
     2 NTH,PR,THA,WAVM)
C      SEPTEMBER 21, 1990.
C     CALCULATE DIRECTIVE GAIN G(N) FOR MONOPOLE ON DISK IN FREE SPACE.
C     ALSO PR = TIME-AVERAGE POWER RADIATED.
C       GAINA = DIRECTIVITY IN FREE SPACE.
C       THA = ANGLE OF MAXIMUM GAIN IN FREE SPACE.
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 CJ(1),CJ1,CJ2,CQ,CQS,EK1,EK2,EK3,ET,ETH,ETHD
      DIMENSION G(1)
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      BET=TP/WAVM
      DKD=(CK-AK)/NSD
      SDKD=DSIN(DKD)
      CDKD=DCOS(DKD)
      DKW=HK/NSW
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      NEQ=NSD+NSW-1
      DK=HK
      IF(CK.GT.HK)DK=CK
      NS=10.*DK/PI
      NS=2*(NS/2)
      IF(NS.LT.20)NS=20
      NTH=NS+1
      DTH=PI/NS
      CQ=DCMPLX(.0D0,BET*ETA/(4.*PI*SDK))
C       Z0=HDK
C       Z0=.0
      LS=10.*DKD/PI
      IF(LS.LT.4)LS=4
      LS=2*(LS/2)
      PR=.0
        GAINA=.0
      DO 250 M=1,2
C     IF M=1, USE SIMPSON'S RULE INTEGRATION TO
C             CALCULATE PR = POWER RADIATED.
C     IF M=2, CALCULATE THE DIRECTIVE GAIN G(N).
      SGN=-1.
      DO 200 NT=1,NTH
      IF(NT.EQ.1)GO TO 200
      IF(NT.EQ.NTH)GO TO 200
      ET=(.0D0,.0D0)
      WF=3.+SGN
C     SELECT THE FAR-FIELD ANGLE TH = THETA.
      TH=DTH*(NT-1)
      CTH=DCOS(TH)
      STH=DSIN(TH)
      CQS=CQ/STH
C     CALCULATE FAR-ZONE FIELD OF MODE #1 IN FREE SPACE.
      Z1=Z0
      Z2=Z1+DKW
      ARG=Z2*CTH
      EK2=DCMPLX(DCOS(ARG),DSIN(ARG))
      ARG=Z1*CTH
      EK1=DCMPLX(CDK,CTH*SDK)*DCMPLX(DCOS(ARG),DSIN(ARG))
      ETH=CQS*CJ(1)*(EK2-EK1)
      CJ1=-CJ(1)
      CJ2=(.0D0,.0D0)
      RK1=AK
      RK2=RK1+DKD
      CALL EDISK1(CJ1,CJ2,CTH,DKD,ETHD,LS,RK1,RK2,SDKD,STH,BET,Z0)
      ET=ET+ETH+ETHD
```

```
      IF(NSD.LE.1)GO TO 100
C     CALCULATE FAR FIELD FROM DISK CURRENT IN FREE SPACE.           GAIN1.2
      DO 60 J=1,NSD
      RK1=AK+(J-1)*DKD
      RK2=RK1+DKD
      CJ1=CJ(J)
      IF(J.EQ.1)CJ1=(.0D0,.0D0)
      CJ2=CJ(J+1)
      IF(J.EQ.NSD)CJ2=(.0D0,.0D0)
C     EDISK1 CALCULATES FIELD FROM ANNULAR ZONE J OF THE DISK.
C     RK1 AND RK2 ARE INNER AND OUTER RADII OF ZONE J.
C     CJ1 AND CJ2 DENOTE RADIAL CURRENTS AT INNER AND OUTER RADII.
      CALL EDISK1(CJ1,CJ2,CTH,DKD,ETHD,LS,RK1,RK2,SDKD,STH,BET,Z0)
      ET=ET+ETHD
   60 CONTINUE
  100 IF(NSW.LE.1)GO TO 162
C     CALCULATE FAR FIELD FROM THE WIRE DIPOLE MODES IN FREE SPACE.
      JB=NSW-1
      DO 160 J=1,JB
      Z2=Z0+J*DKW
      Z1=Z2-DKW
      Z3=Z2+DKW
      L=NSD+J
      ARG=Z1*CTH
      EK1=DCMPLX(DCOS(ARG),DSIN(ARG))
      ARG=Z2*CTH
      EK2=DCMPLX(DCOS(ARG),DSIN(ARG))
      ARG=Z3*CTH
      EK3=DCMPLX(DCOS(ARG),DSIN(ARG))
      ETH=CJ(L)*CQS*(EK1+EK3-2.*CDK*EK2)
      ET=ET+ETH
  160 CONTINUE
  162 AT=CDABS(ET)
      ATS=AT**2
      IF(M.EQ.1)PR=PR+WF*ATS*STH
      IF(M.EQ.2)G(NT)=ATS
        IF(ATS.LT.GAINA)GO TO 200
        GAINA=ATS
        THA=57.29578*TH
  200 SGN=-SGN
      IF(M.EQ.1)PR=TP*PR*DTH/(3.*ETA*BET*BET)
      DTH=.01745329*DTHD
        IF(DTHD.GT.0.)GO TO 248
        NS=1
        NTH=1
        GO TO 250
  248   NS=180./DTHD
      NTH=NS+1
  250 CONTINUE
      G(1)=.0
      G(NTH)=.0
C     NORMALIZE THE DIRECTIVE GAIN G(N).
      CST=4.*PI/(ETA*BET*BET*PR)
        GAINA=CST*GAINA
      DO 300 N=1,NTH
      GN=CST*G(N)
      G(N)=GN
  300 CONTINUE
      RETURN
      END
```

```
C
C
        SUBROUTINE GAIN2(AK,CK,CJ,DTHD,EC,G,GAINB,HDK,HK,         GAIN2.1
     2  NSD,NSW,NTH,PR,THB,WAVM)
C       SEPT. 21, 1990.
C       GAINB = DIRECTIVITY OVER FLAT EARTH.
C       THB = ANGLE OF MAXIMUM GAIN.
C     CALCULATE DIRECTIVE GAIN G(N) FOR MONOPOLE ON DISK OVER FLAT EARTH.
C     ALSO PR = TIME-AVERAGE POWER RADIATED INTO UPPER HALF-SPACE.
C     DOUBLE PRECISION
      IMPLICIT REAL*8 (A-H), (P-Z)
      COMPLEX*16 CJ(1),CJ1,CJ2,CQ,CQS
      COMPLEX*16 EC,EK1,EK2,EK3,ET,ETH,ETHD,ET1,QST,RC
      DIMENSION G(1)
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      BET=TP/WAVM
      DKD=(CK-AK)/NSD
      SDKD=DSIN(DKD)
      CDKD=DCOS(DKD)
      DKW=HK/NSW
      CDK=DCOS(DKW)
      SDK=DSIN(DKW)
      NEQ=NSD+NSW-1
      DK=HK+HDK
      IF(CK.GT.DK)DK=CK
      NS=10.*DK
      NS=2*(NS/2)
      IF(NS.LT.90)NS=90
      NTH=NS+1
      DTH=PI/(2.*NS)
      CQ=DCMPLX(.0D0,BET*ETA/(4.*PI*SDK))
      Z0=HDK
      LS=10.*DKD/PI
      IF(LS.LT.4)LS=4
      LS=2*(LS/2)
      PR=.0
        GAINB=.0
      DO 250 M=1,2
C     IF M=1, USE SIMPSON'S RULE INTEGRATION TO
C              CALCULATE PR = POWER RADIATED.
C     IF M=2, CALCULATE DIRECTIVE GAIN G(N).
      SGN=-1.
      DO 200 NT=1,NTH
      IF(NT.EQ.1)GO TO 200
      ET=(.0D0,.0D0)
      WF=3.+SGN
      IF(NT.EQ.NTH)WF=1.
C     SELECT THE FAR-FIELD ANGLE TH = THETA.
C
      TH=DTH*(NT-1)
      CTH=DCOS(TH)
      STH=DSIN(TH)
      CQS=CQ/STH
      QST=CDSQRT(EC-STH*STH)
C     RC = REFLECTION COEFFICIENT AT AIR-EARTH INTERFACE.
C
      RC=(EC*CTH-QST)/(EC*CTH+QST)
C     CALCULATE FAR-ZONE FIELD OF MODE #1.
C
      Z1=Z0
      Z2=Z1+DKW
      ARG1=Z1*CTH
      EK1=DCMPLX(DCOS(ARG1),DSIN(ARG1))
      ET1=DCMPLX(CDK,CTH*SDK)*EK1
      ARG2=Z2*CTH
      EK2=DCMPLX(DCOS(ARG2),DSIN(ARG2))
```

```
      ETH=EK2-ET1
      EK1=DCONJG(EK1)
      ET1=DCMPLX(CDK,-CTH*SDK)*EK1
      EK2=DCONJG(EK2)
      ETH=CQS*CJ(1)*(ETH+RC*(EK2-ET1))
      CJ1=-CJ(1)
      CJ2=(.0D0,.0D0)
      RK1=AK
      RK2=RK1+DKD
      CALL EDISK2(CJ1,CJ2,CTH,DKD,ETHD,LS,RC,RK1,RK2,SDKD,STH,BET,Z0)
      ET=ET+ETH+ETHD
      IF(NSD.LE.1)GO TO 100
C     CALCULATE FAR FIELD FROM DISK CURRENT IN FREE SPACE.
C
      DO 60 J=1,NSD
      RK1=AK+(J-1)*DKD
      RK2=RK1+DKD
      CJ1=CJ(J)
      IF(J.EQ.1)CJ1=(.0D0,.0D0)
      CJ2=CJ(J+1)
      IF(J.EQ.NSD)CJ2=(.0D0,.0D0)
C     EDISK2 CALCULATES FIELD FROM ANNULAR ZONE J OF THE DISK.
C     RK1 AND RK2 ARE INNER AND OUTER RADII OF ZONE J.
C     CJ1 AND CJ2 DENOTE RADIAL CURRENTS AT INNER AND OUTER RADII.
C
      CALL EDISK2(CJ1,CJ2,CTH,DKD,ETHD,LS,RC,RK1,RK2,SDKD,STH,BET,Z0)
      ET=ET+ETHD
   60 CONTINUE
  100 IF(NSW.LE.1)GO TO 162
C     CALCULATE FAR FIELD FROM THE WIRE DIPOLE MODES IN FREE SPACE.
C
      JB=NSW-1
      DO 160 J=1,JB
      Z2=Z0+J*DKW
      Z1=Z2-DKW
      Z3=Z2+DKW
      L=NSD+J
      ARG=Z1*CTH
      EK1=DCMPLX(DCOS(ARG),DSIN(ARG))
      ARG=Z2*CTH
      EK2=DCMPLX(DCOS(ARG),DSIN(ARG))
      ARG=Z3*CTH
      EK3=DCMPLX(DCOS(ARG),DSIN(ARG))
      ETH=EK1+EK3-2.*CDK*EK2
      EK1=DCONJG(EK1)
      EK2=DCONJG(EK2)
      EK3=DCONJG(EK3)
      ETH=CJ(L)*CQS*(ETH+RC*(EK1+EK3-2.*CDK*EK2))
      ET=ET+ETH
  160 CONTINUE
  162 AT=CDABS(ET)
      ATS=AT**2
      IF(M.EQ.1)PR=PR+WF*ATS*STH
      IF(M.EQ.2)G(NT)=ATS
        IF(ATS.LT.GAINB)GO TO 200
        GAINB=ATS
        THB=57.29578*TH
  200 SGN=-SGN
      IF(M.EQ.1)PR=TP*PR*DTH/(3.*ETA*BET*BET)
        IF(DTHD.GT.0.)GO TO 248
        NS=1
        NTH=1
        GO TO 250
  248   DTH=.01745329*DTHD
      NS=90./DTHD
      NTH=NS+1
```

```
  250 CONTINUE
      G(1)=.0                                                   GAIN2.3
C     NORMALIZE THE DIRECTIVE GAIN G(N).
      CST=4.*PI/(ETA*BET*BET*PR)
        GAINB=CST*GAINB
      DO 300 N=1,NTH
      GN=CST*G(N)
      G(N)=GN
  300 CONTINUE
      RETURN
      END
C
C
```

APPENDIX C

COMPUTER PROGRAM RICHMD6 FOR THE INPUT IMPEDANCE, CURRENT DISTRIBUTION, AND FAR-ZONE FIELD OF A MONOPOLE ELEMENT ON A PERFECT GROUND PLANE

```
C            PROGRAM "RICHMD6.FOR"
C***************************************************************************
C                                                                         *
C                                                                         *
C        THIS COMPUTER PROGRAM, IN FORTRAN LANGUAGE, WAS WRITTEN BY DR.   *
C   JACK RICHMOND OF OHIO STATE UNIVERSITY.  IT USES A SINUSOIDAL-GALERKIN *
C   METHOD OF MOMENTS TO COMPUTE THE INPUT IMPEDANCE, CURRENT DISTRIBUTIONS, *
C   AND ANTENNA PATTERN OF A MONOPOLE ELEMENT OF LENGTH h AND RADIUS b ON A *
C   PERFECT GROUND PLANE OF INFINITE EXTENT.  A DETAILED DERIVATION IS    *
C   PUBLISHED IN REFERENCE 1.                                             *
C                                                                         *
C                                                                         *
C        REFERENCE:                                                       *
C                                                                         *
C        1)   J.H. RICHMOND, "COMPUTER PROGRAM WAIT-SURTEES", REPORT      *
C                  PREPARED FOR THE MITRE CORPORATION, 29 DEC. 1989.      *
C                                                                         *
C***************************************************************************
C                                                                         *
C        THIS COMPUTER PROGRAM REQUIRES FIVE INPUTS WHICH ARE ENTERED FROM *
C   AN INPUT FILE NAMED "RICH6_IN.DAT".                                   *
C                                                                         *
C                                                                         *
C                  b/λ = AL = MONOPOLE ELEMENT RADIUS IN WAVELENGTHS      *
C                  h/λ = HL = MONOPOLE ELEMENT LENGTH IN WAVELENGTHS      *
C                 b1/b = BAR = RATIO OF OUTER TO INNER CONDUCTOR RADII OF *
C                              THE COAXIAL LINE FEED.                     *
C                   σw = CMM = CONDUCTIVITY OF MONOPOLE ELEMENT           *
C                              (MEGAMHOS/METER)                           *
C                            = -1 FOR PERFECTLY CONDUCTING MONOPOLE ELEMENT *
C                IFLAG = FLAG FOR MONOPOLE ELEMENT CURRENT DISTRIBUTIONS  *
C                      = 0, MONOPOLE ELEMENT CURRENTS COMPUTED BY METHOD OF *
C                           MOMENTS                                       *
C                      = -1, MONOPOLE ELEMENT CURRENTS WITH IDEALIZED     *
C                            SINUSOIDAL DISTRIBUTION AND FOR INFINITE     *
C                            CONDUCTIVITY OF THE MONOPOLE ELEMENT         *
C                    f = FMC = FREQUENCY IN MEGAHERTZ                     *
C                              NOTE: IF CMM = -1, IT IS NOT NECESSARY TO  *
C                                    SPECIFY AN INPUT VALUE FOR FREQUENCY *
C                                                                         *
C                                                                         *
C***************************************************************************
C
C     LINK CISI;FRILLS;TPLZ;TSPAR;ZSURF
      REAL G(100),ANG(100),DIRECDB(100),DIRECRP(100),DIRMAX
      INTEGER HALFNTH,NTH
      COMPLEX C00,CJI,CJA,CJB,DELZ
      COMPLEX EC,EGZ,EJH,EJS,ETA2,ETD,ETH
      COMPLEX GM,GP,Q1,Q2,Q3,RC
      COMPLEX Y11,Z11,ZFIN,ZINF,ZS,ZSG
      COMPLEX CJ(99),GI(99),U(99),VJ(99),W(99),ZJ(99)
      OPEN(UNIT=4,FILE='RICH6_IN.DAT',STATUS='OLD',READONLY)
      OPEN(UNIT=10,FILE='RICH6_OUT.DAT',STATUS='NEW')
      DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
      DATA E0,U0/8.85418533677E-12,1.25663706144E-6/
      DATA P2,E/1.57079632679,2.718281828/
      DATA IDM/99/
C**************************************************************************
C     WAVM    = WAVELENGTH IN FREE-SPACE (METERS).                       *
C               NOTE: NOT USED IF CMM = -1.                              *
```

```
C*****************************************************************************
  2   FORMAT(7X,8F11.5)
  3   FORMAT(8X,8F16.10)
  4   FORMAT(4X,8F11.5)
  7   FORMAT(5X)
  521 FORMAT(1X,A)
C*****************************************************************************
C          INPUTS                                                            *
C*****************************************************************************
      READ(4,*)AL,HL,BAR,CMM,IFLAG
      IF(CMM.EQ.-1)GOTO 8
      READ(4,*) FMC
      WAVM=300./FMC
  8   CONTINUE
      IWR=1
C*****************************************************************************
C          INPUT WRITE STATEMENTS                                            *
C*****************************************************************************
      CALL HEADING2(AL,HL,BAR,CMM,IFLAG)
      AK=TP*AL
      HK=TP*HL
      SK=2.*HK
      EJS=CMPLX(COS(SK),SIN(SK))
      CHK=COS(HK)
      SHK=SIN(HK)
      EJH=CMPLX(CHK,SHK)
      NS=15.*HL
      IF(NS.LT.6)NS=6
      IF(NS.GT.IDM)NS=IDM
      IG=NS/2
      IF(IFLAG.EQ.0)GOTO 43
      IG=1
      CMM=-1.
  43  CONTINUE
      NS=2*IG
      N=NS-1
      ZS=(.0,.0)
      IF(CMM.GT.0.)CALL ZSURF(AK,CMM,FMC,ZS)
C*****************************************************************************
C     ZS = SURFACE IMPEDANCE OF MONOPOLE WIRE.                               *
C*****************************************************************************
      DK=HK/IG
      CDK=COS(DK)
      SDK=SIN(DK)
C*****************************************************************************
C     ZJ(N) = FIRST ROW OF IMPEDANCE MATRIX FOR DIPOLE IN FREE SPACE,        *
C     USING GALERKIN'S METHOD WITH OVERLAPPING SINUSOIDAL BASIS FUNCTIONS.   *
C     (DIPOLE LENGTH = TWICE THE MONOPOLE LENGTH, USING IMAGE THEORY.)       *
C*****************************************************************************
      CALL TSPAR(AK,DK,N,ZJ)
      IF(CMM.LT.0.)GO TO 52
      GK=2.*(DK-CDK*SDK)
      GL=SDK-DK*CDK
      FH=4.*PI*AK*SDK*SDK
      ZJ(1)=ZJ(1)+ZS*GK/FH
      ZJ(2)=ZJ(2)+ZS*GL/FH
  52  I12=1
C*****************************************************************************
C     SET UP THE MOMENT-METHOD VOLTAGE COLUMN VJ(M) FOR DIPOLE IN FREE SPACE.*
C*****************************************************************************
```

```
      CALL FRILLS(AK,BAR,DK,N,CJ)
      DO 60 I=1,N
      K=1+IABS(IG-I)
   60 VJ(I)=CJ(K)
C*****************************************************************************
C     SOLVE THE SIMULTANEOUS LINEAR EQUATIONS TO DETERMINE THE CURRENTS CJ(N)*
C     ON THE WIRE DIPOLE IN FREE SPACE.  (THE IMPEDANCE MATRIX IS TOEPLITZ.) *
C*****************************************************************************
      CALL PRCURR
      CALL TPLZ(CJ,U,VJ,V,ZJ,IER,IWR,I12,N)
      WRITE(10,521)CHAR(12)
C*****************************************************************************
C     ZINF = IMPEDANCE OF MONOPOLE ANTENNA OVER INFINITE GROUND PLANE.       *
C*****************************************************************************
      CALL HEADING2(AL,BL,BAR,CMM,IFLAG)
      ZINF=.5/CJ(IG)
      RIN=REAL(ZINF)
      XIN=AIMAG(ZINF)
      CALL PRRES(RRAD,RIN,XIN)
      Y11=CJ(IG)
      PD=.0
      PIN=REAL(Y11)
      PRAD=PIN-PD
      EFF=100.*PRAD/PIN
      DTH=2.
      NTH=1.+90./DTH

      DO I=1,NTH
        ANG(I)=(I-1)*DTH
        TH=ANG(I)
        CALL GAIN(CDK,CJ,DGAIN,DK,U,HK,N,PRAD,SDK,TH,VJ,WAVM)
        G(I)=DGAIN*2
      END DO

      DO I=(NTH+1),91
        G(I)=0
        ANG(I)=(I-1)*DTH
      END DO

C      DO I=1,NTH
C        WRITE(10,2)ANG(I),G(I),ANG(I+NTH),G(I+NTH)
C      END DO

C*****************************************************************************
C   OUTPUT STATEMENTS FOR FAR-FIELD PATTERN HEADINGS  (WITH EARTH)           *
C*****************************************************************************
      WRITE(10,850)
  850 FORMAT(//1X,
     &   'ELEVATION',5X,'DIRECTIVE',6X,'DIRECTIVE',5X,'RELATIVE',12X,
     &   'ELEVATION',5X,'DIRECTIVE',6X,'DIRECTIVE',5X,'RELATIVE' )
      WRITE(10,852)
  852 FORMAT(3X,'ANGLE',9X,'GAIN',11X,'GAIN',9X,'POWER',
     &       16X,'ANGLE',9X,'GAIN',11X,'GAIN',9X,'POWER' )
      WRITE(10,854)
  854 FORMAT(3X,'(DEG)',7X,'(NUMERIC)',8X,'(DBI)',9X,'(DB)',
     &       16X,'(DEG)',7X,'(NUMERIC)',8X,'(DBI)',9X,'(DB)',/5X )
C*****************************************************************************
C*****************************************************************************
      DIRMAX=0.0
      DO N=1,NTH
```

```
        IF(G(N).NE.0)DIRECDB(N)=10.0*LOG10(G(N))
        IF(DIRMAX.LT.DIRECDB(N))DIRMAX=DIRECDB(N)
      END DO
      DO N=1,NTH
        DIRECRP(N)=DIRECDB(N)-DIRMAX
      END DO
C*******************************************************************************
C   OUTPUT STATEMENTS FOR FAR FIELD-PATTERN  (WITH EARTH)                      *
C*******************************************************************************

      HALFNTH = NTH/2

      DO N=1,HALFNTH
        IF( G(N).NE.0) THEN
          WRITE(10,750)ANG(N),G(N),DIRECDB(N),DIRECRP(N),
     &                ANG(N+HALFNTH),G(N+HALFNTH),DIRECDB(N+HALFNTH),
     &                DIRECRP(N+HALFNTH)
  750     FORMAT(3X,F4.0,2F16.5,F13.5,14X,F4.0,2F16.5,F13.5)
        ELSE
          WRITE(10,604)ANG(N),ANG(N+HALFNTH),G(N+HALFNTH),
     &                 DIRECDB(N+HALFNTH),DIRECRP(N+HALFNTH)
  604     FORMAT(3X,F4.0,9X,'0.00000',7X,'-INFINITY',4X,'-INFINITY',
     &           14X,F4.0,2F16.5,F13.5 )
        END IF
      END DO

  320 CONTINUE
C*******************************************************************************
C*******************************************************************************

      CLOSE(UNIT=4,STATUS='KEEP')
      CLOSE(UNIT=10,STATUS='KEEP')
  400 CALL EXIT
      END
C
C
C*****************************************************************************
C          SUBROUTINE GAIN                                                   *
C*****************************************************************************
      SUBROUTINE GAIN(CDK,CJ,DGAIN,DK,ETT,HK,N,PRAD,SDK,TH,VJ,
     &                WAVM)
C
       COMPLEX CJ(1),ETT(1),VJ(1),CQQ,ETH,ETD
       DATA ETA/376.727/

       CQQ=CMPLX(.0,2./ETA)
       CTH=COS(.0174533*TH)
       STH=SIN(.0174533*TH)
       ETD=(.0,.0)
       IF(TH.GT.1.)
     &   ETD=-60.*(.0,1.)*(CDK-COS(DK*CTH))/(SDK*STH)
       Z=-HK+DK
       ETH=(.0,.0)
       DO 125 I=1,N
         ANG=Z*CTH
         ETT(I)=ETD*CMPLX(COS(ANG),SIN(ANG))
         VJ(I)=CQQ*ETT(I)*WAVM
         IF(PRAD.GT..0)ETH=ETH+CJ(I)*ETT(I)
         IF(PRAD.LT..0)CJ(I)=VJ(I)
  125  Z=Z+DK
```

```
      IF(PRAD.GT..0)DGAIN=(CABS(ETH)**2)/(30.*PRAD)
      RETURN
      END

      SUBROUTINE PRCURR
C***********************************************************************
C     OUTPUT STATEMENTS FOR CURRENT DISTRIBUTIONS                     *
C***********************************************************************
      WRITE(10,502)
  502 FORMAT(///17X,'CURRENT DISTRIBUTIONS ON MONOPOLE')
      WRITE(10,506)
  506 FORMAT(/5X,'I',12X,'CJ(I)',15X,'CJ(I)',14X,'CJ(I)')
      WRITE(10,508)
  508 FORMAT(17X,'(NORM)',15X,'(MAG)',8X,'(PHASE IN DEGREES)',5X)
C***********************************************************************
C***********************************************************************
      RETURN
      END
C
C
      SUBROUTINE PRRES(RRAD,RI,XI)
C
C***********************************************************************
C   OUTPUT STATEMENTS FOR RADIATION RESISTANCE, INPUT IMPEDANCE, AND  *
C     RADIATION EFFICIENCY                                            *
C***********************************************************************
C
      REAL*8 ETA
C
      WRITE(10,954)RRAD
  954 FORMAT(1X,'RADIATION RESISTANCE IN OHMS, Rrad',
     &       ' (by integration of radiation pattern)',
     &       5X,'Rrad = ',F10.4 )

      WRITE(10,952)RI,XI
  952 FORMAT(1X,'INPUT IMPEDANCE IN OHMS, Rin + jXin',
     &       42x,'Rin = ',F11.4,
     &       /77X,' Xin = ',F11.4 )
      ETA=RRAD/RI
      WRITE(10,956)ETA
  956 FORMAT(1X,'RADIATION EFFICIENCY, ETA = Rrad/Rin ',
     &       40X,'ETA = ',F10.3)
C***********************************************************************
C***********************************************************************
      RETURN
      END
C
C
C
      SUBROUTINE EARTHTYPE
C***********************************************************************
C     PRINTS OUT THE PROGRAM NAME AND EARTH TYPE                      *
C***********************************************************************
C
C
        INTEGER CASE
        CHARACTER*40 TYPEEARTH

        TYPEEARTH = ' PERFECT GROUND'
        CASE = 1
```

```
  1     WRITE(10,100)CASE,TYPEEARTH
 100    FORMAT(///21X,'PROGRAM RICHMD6',5X,'CASE NO. ',I2,',',(A))
C*******************************************************************************
C*******************************************************************************
        RETURN
        END
C
C
C
      SUBROUTINE HEADING2(AL,HL,BAR,CMM,IFLAG)
C     PRINTS THE HEADINGS FOR OVER FLAT LOSSY EARTH
C
C
C*******************************************************************************
C          PROGRAM DESCRIPTION AND ECHOED INPUT (WITH EARTH)                   *
C*******************************************************************************
C
      CALL EARTHTYPE
      WRITE(10,224)AL
 224  FORMAT(/1X,'MONOPOLE ELEMENT RADIUS IN WAVELENGTHS, AL',35X,
     A      'AL = ',F18.10)
      WRITE(10,226)HL
 226  FORMAT(1X,'MONOPOLE ELEMENT LENGTH IN WAVELENGTHS, HL',35X,
     A      'HL = ',F18.10)
      WRITE(10,228)
 228  FORMAT(1X,'RATIO OF OUTER TO INNER CONDUCTOR RADII OF THE',
     A      ' COAXIAL')
      WRITE(10,230)BAR
 230  FORMAT(1X,' LINE FEED, BAR',62X,'BAR = ',F10.3)
      WRITE(10,83)
  83  FORMAT(1X,'CONDUCTIVITY OF MONOPOLE ELEMENT (MEGAMHOS/METER)',
     A      ', CMM ')
      WRITE(10,84)CMM
  84  FORMAT(1X,' = -1 FOR PERFECTLY CONDUCTING MONOPOLE ELEMENT',
     A      30X,'CMM = ',F10.3)
      WRITE(10,36)
  36  FORMAT(1X,'FLAG FOR MONOPOLE ELEMENT CURRENT DISTRIBUTIONS',
     A      ', IFLAG')
      WRITE(10,37)
  37  FORMAT(1X,' = 0, MONOPOLE ELEMENT CURRENTS COMPUTED BY METHOD',
     A      ' OF')
      WRITE(10,11)
  11  FORMAT(7X,'MOMENTS')
      WRITE(10,38)
  38  FORMAT(1X,' = -1, MONOPOLE ELEMENT CURRENTS WITH IDEALIZED',
     A      ' SINUSOIDAL')
      WRITE(10,12)
  12  FORMAT(8X,'DISTRIBUTION AND FOR INIFINITE CONDUCTIVITY OF THE')
      WRITE(10,13)IFLAG
  13  FORMAT(8X,'MONOPOLE ELEMENT',54X,'IFLAG = ',I4)
      IF(CMM.EQ.-1)GOTO 88
      WRITE(10,86)FMC
  86  FORMAT(1X,'FREQUENCY IN MEGAHERTZ, FMC',41X,'FMC = ',F8.3)
  88  CONTINUE

C
C*******************************************************************************
C*******************************************************************************
      RETURN
```

```
      END
C
C
C
C*****************************************************************************
C         SUBROUTINE CISI                                                     *
C*****************************************************************************
      SUBROUTINE CISI(CI,CIN,SI,X)
C*****************************************************************************
C     Standard IBM Fortran Subroutine with slight modifications.             *
C     COSINE INTEGRAL AND SINE INTEGRAL.                                      *
C     X = ARGUMENT (REAL AND POSITIVE).                                       *
C     CI = Ci(x).                                                             *
C     SI = Si(x).                                                             *
C     CIN = Cin(x).                                                           *
C*****************************************************************************
      DATA GAM,P2/.57721566,1.57079632/
      A=ABS(X)
      IF(A.GT.4.)GO TO 10
      IF(A.GT..1)GO TO 3
      IF(A.GT.0.)GO TO 2
      CI=.0
      CIN=.0
      SI=.0
      RETURN
  2   X2=A*A
      SI=X*((.03*X2-1.)*X2/18.+1.)
      CIN=.25*X2*((X2/45.-1.)*X2/24.+1.)
      GO TO 8
  3   Y=(4.-A)*(4.+A)
      SI=X*(((((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)*Y+6.939889E-4)
     C*Y+1.964882E-2)*Y+4.395509E-1)
      CIN=                     A*A*(((((1.386985E-10*Y+1.584996E-8)*Y
     C+1.725752E-6)*Y+1.185999E-4)*Y+4.990920E-3)*Y+1.315308E-1)
  8   CI=GAM+ALOG(A)-CIN
      RETURN
 10   SI=SIN(A)
      Y=COS(A)
      Z=4./A
      U=((((((((4.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)*Z-7.261642E-2)
     C*Z+4.987716E-2)*Z-3.332519E-3)*Z-2.314617E-2)*Z-1.134958E-5)*Z
     C+6.250011E-2)*Z+2.583989E-10
      V=(((((((((-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z
     C+7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+2.601293E-2)*Z
     C-3.764000E-4)*Z-3.122418E-2)*Z-6.646441E-7)*Z+2.500000E-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)+P2
      IF(X.LT..0)SI=-SI
      CIN=GAM+ALOG(A)-CI
      RETURN
      END
C
C
C*****************************************************************************
C         SUBROUTINE FRILLS                                                   *
C*****************************************************************************
      SUBROUTINE FRILLS(AK,BAR,DK,NEQ,VJ)
C*****************************************************************************
C     FRILLS sets up the voltage column VJ(I).                                *
C     VJ(I) = voltage column for perfectly conducting wire dipole in         *
```

```
C      free space using Galerkin's method and sinusoidal bases, and         *
C      matching the boundary conditions on the surface of the wire.         *
C     Using magnetic-frill model for center-fed dipole.                     *
C****************************************************************************
      REAL*8 DZS,RS1,RS2
      COMPLEX EGZ,GM,GP,GI(20),VJ(1),GII,QST,WST
      DATA PI,TP/3.14159265359,6.28318530718/
      IDM=20
      DO 20 I=1,NEQ
   20 VJ(I)=(.0,.0)
      VJ(1)=(1.,.0)
      IF(BAR.LE.1.)RETURN
      VJ(1)=(.0,.0)
      NSW=NEQ+1
      SDK=SIN(DK)
      CDK=COS(DK)
      BAL=ALOG(BAR)
      QST=CMPLX(.0,1./(4.*BAL*SDK))
      BK=AK*BAR
      AKS=AK*AK
      BKS=BK*BK
      LIM=NSW+1
      IF(LIM.GT.IDM)LIM=IDM
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      PH=.0
      DO 90 LPH=1,2
      WST=DPH*QST/(3.*PI)
      SGN=-1.
      DO 80 IPH=1,NPP
      WF=3.+SGN
      IF(IPH.EQ.1)WF=1.
      IF(IPH.EQ.NPP)WF=1.
      CPH=COS(PH)
      IF(IPH.GT.1)GO TO 40
      IF(LPH.GT.1)GO TO 40
      CPH=COS(DPH/10.)
   40 RS1=2.*AKS*(1.-CPH)
      RS2=AKS+BKS-2.*AK*BK*CPH
      RH1=DSQRT(RS1)
      RH2=DSQRT(RS2)
      CALL CISI(CA,CIN,SA,RH1)
      CALL CISI(CB,CIN,SB,RH2)
      GI(1)=2.*CMPLX(CB-CA,SA-SB)
      DO 50 I=2,LIM
      DZ=DK*(I-1)
      DZS=DZ*DZ
      RA=DSQRT(RS1+DZS)
      RB=DSQRT(RS2+DZS)
      CALL CISI(C1,CIN,S1,RA+DZ)
      CALL CISI(C2,CIN,S2,RB+DZ)
      GP=CMPLX(C2-C1,S1-S2)
      RAM=RS1/(RA+DZ)
      RBM=RS2/(RB+DZ)
      CALL CISI(C1,CIN,S1,RAM)
      CALL CISI(C2,CIN,S2,RBM)
      GM=CMPLX(C2-C1,S1-S2)
```

```
      EGZ=CMPLX(COS(DZ),SIN(DZ))
   50 GI(I)=GP*EGZ+GM/EGZ
      VJ(1)=VJ(1)+WF*WST*(GI(2)-CDK*GI(1))
      IF(NEQ.LE.1)GO TO 78
      K1=0
      DO 60 I=2,NEQ
      K1=K1+1
      K2=K1+1
      K3=K2+1
      IF(K3.GT.IDM)GO TO 60
      GP=GI(K1)-2.*CDK*GI(K2)+GI(K3)
      VJ(I)=VJ(I)+WF*WST*GP
   60 CONTINUE
   78 SGN=-SGN
   80 PH=PH+DPH
      DPH=(PI-PHA)/NPH
   90 PH=PHA
      VJ(1)=2.*VJ(1)
      RETURN
      END
C
C
C*****************************************************************************
C          SUBROUTINE TPLZ                                                   *
C*****************************************************************************
      SUBROUTINE TPLZ(C,U,VJ,W,Z,IER,IWR,I12,NEQ)
C*****************************************************************************
C       MODIFIED VERSION OF SUBROUTINE FURNISHED BY CHARLES KLEIN.           *
C       SOLVES SIMULTANEOUS LINEAR EQUATIONS.                                *
C       SET IWR =(1 OR 0) TO GET (PRINTOUT OR NO-PRINTOUT).                  *
C       SET I12 = 1 ON FIRST CALL, WHERE MATRIX INVERSION IS REQUIRED.       *
C       SET I12 = 2 IF MATRIX Z HAS ALREADY BEEN INVERTED ON PREVIOUS CALL.  *
C       NEQ = NUMBER OF SIMULTANEOUS LINEAR EQUATIONS.                       *
C  Z(J) IS THE FIRST ROW OF THE TOEPLITZ IMPEDANCE MATRIX                    *
C  VJ(J) = INPUT VOLTAGE COLUMN                                              *
C  C(J) = OUTPUT CURRENT COLUMN                                              *
C  U(J) AND W(J) ARE WORK ARRAYS OF LENGTH NEQ                               *
C  IF IER = 0 , NO ERROR OCCURRED                                            *
C*****************************************************************************
      COMPLEX C(1),U(1),VJ(1),W(1),Z(1)
      COMPLEX ALMDA,ALPHA,C1,C2,COEF,FAC,TAU1,V,V1,V2
    2 FORMAT(1X,I5,7X,F10.4,7X,F15.7,7X,F10.1)
    7 FORMAT(5X)
      IF(NEQ.GT.1)GO TO 8
      C(1)=VJ(1)/Z(1)
      CNOR=CABS(C(1))
      GO TO 100
    8 IF(I12.NE.1)GO TO 45
      N=NEQ-1
      IER=0
C*****************************************************************************
C  NORMALIZE INPUT MATRIX                                                    *
C*****************************************************************************
      TAU1=Z(1)
      DO 10 II=1,N
   10 Z(II)=Z(II+1)/TAU1
      ALMDA=1.-Z(1)*Z(1)
      U(1)=-Z(1)
      I=2
   15 KK=I-1
```

```
      ALPHA=(.0,.0)
      DO 20 M=1,KK
      LL=I-M
   20 ALPHA=ALPHA+U(M)*Z(LL)
      ALPHA=-(ALPHA+Z(I))
      IF(CABS(ALPHA).EQ..0)GO TO 130
      COEF=ALPHA/ALMDA
      ALMDA=ALMDA-COEF*ALPHA
      DO 30 J=1,KK
      L=I-J
   30 V(J)=U(J)+COEF*U(L)
      DO 40 J=1,KK
   40 U(J)=V(J)
      U(I)=COEF
      IF(I.GE.N)GO TO 45
      I=I+1
      GO TO 15
C***************************************************************************
C  THE FOLLOWING COMPUTES THE ELEMENTS OF THE INVERSE                      *
C***************************************************************************
   45 NH=(NEQ+1)/2
      FAC=ALMDA*TAU1
      NP=NEQ+1
      CNOR=.0
      DO 90 I=1,NH
      IF(I.NE.1)GO TO 55
      V(1)=1./FAC
      DO 50 J=2,NEQ
   50 V(J)=U(J-1)/FAC
      GO TO 70
   55 C1=U(I-1)
      NPI=NP-I
      C2=U(NPI)
      DO 60 JJ=1,N
      J=NP-JJ
      NPJ=NP-J
   60 V(J)=V(J-1)+(C1*U(J-1)-C2*U(NPJ))/FAC
      V(1)=U(I-1)/FAC
C***************************************************************************
C  MATRIX MULTIPLY                                                         *
C***************************************************************************
   70 V=(.0,.0)
      V1=(.0,.0)
      DO 80 J=1,NEQ
      V2=VJ(J)
      V=V+V2*V(J)
      NPJ=NP-J
   80 V1=V1+V2*V(NPJ)
      C(I)=V
      NPI=NP-I
      C(NPI)=V1
      IF(IWR.LE.0)GO TO 90
      CA=CABS(V)
      IF(CA.GT.CNOR)CNOR=CA
      CA=CABS(V1)
      IF(CA.GT.CNOR)CNOR=CA
   90 CONTINUE
  100 IF(IWR.LE.0)GO TO 120
C***************************************************************************
C  PRINT OUT THE SOLUTION FOR THE CURRENTS  C(J)                           *
```

```
C*****************************************************************************
      IF(CNOR.LE.0.)CNOR=1.
      DO I=1,(NEQ+1)/2
         V=C(I)
         CA=CABS(V)
         CN=CA/CNOR
         PH=.0
         IF(CA.GT.0.)PH=57.29578*ATAN2(AIMAG(V),REAL(V))
         IF(I.NE.((NEQ+1)/2))WRITE(10,2)I,CN,(CA*2),PH
         IF(I.EQ.((NEQ+1)/2))WRITE(10,22)I,CN,(CA*2),PH
 22      FORMAT(1X,I5,' (base)',f10.4,7x,f15.7,7x,f10.1)
      END DO
 120  RETURN
 130  IER=I
      RETURN
      END
C
C
C*****************************************************************************
C          SUBROUTINE TSPAR                                                  *
C*****************************************************************************
      SUBROUTINE TSPAR(AK,DK,NEQ,Z)
C*****************************************************************************
C     TSPAR sets up the impedance matrix Z(J).                               *
C     Z(J) = First row of impedance matrix for perfectly conducting          *
C       thin-wire dipole in free space, using Galerkin's method with         *
C       overlapping sinusoidal basis functions and matching the              *
C       boundary conditions on the surface of the wire.                      *
C     AK = k*a, where k = 2*pi/lambda and a = wire radius.                   *
C     DK = k*d, where d = segment length.                                    *
C     NEQ = number of simultaneous linear equations.                         *
C*****************************************************************************
      REAL*8 DZS,RS
      COMPLEX EID(90),EM(90),EP(90),Z(1)
      COMPLEX CEM,CEP,EMD,EPD,EMD2,EPD2,Z11,Z22,G11,Q11
      DIMENSION CID(90),SID(90),CM(90),CP(90),SM(90),SP(90)
      DATA GAM,P2/.577215664,1.57079632/
      DATA ETA,PI/376.727,3.14159/
      IDM=90
  1   FORMAT(3X,'MUST INCREASE DIMENSIONS IN SUBROUTINE TSPAR')
  2   FORMAT(3X,'ACTUAL DIMENSION IDM = ',I5,6X,
     2'REQUIRED DIMENSION MAX2 = ',I5)
      IF(NEQ.LE.0)RETURN
      MAX2=NEQ+2
      DO 14 I=1,NEQ
 14   Z(I)=(.0,.0)
      IF(MAX2.LE.IDM)GO TO 16
      WRITE(10,1)
      WRITE(10,2)IDM,MAX2
      RETURN
 16   TDK=2.*DK
      S11=.0
      S13=TDK
      S21=DK
      S23=3.*DK
      DO 20 N=1,MAX2
      I=N-1
      DZ=I*DK
      CID(N)=COS(DZ)
      SID(N)=SIN(DZ)
```

```
 20   EID(N)=CMPLX(CID(N),SID(N))
      CDK=COS(DK)
      SDK=SIN(DK)
      EPD=CMPLX(CDK,SDK)
      EMD=CMPLX(CDK,-SDK)
      EPD2=EPD*EPD
      EMD2=EMD*EMD
      CEM=2.*CDK+EMD
      CEP=2.*CDK+EPD
      AK2=AK*AK
      CSS=ETA/(8.*PI*SDK*SDK)
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      PH=.0
      DO 100 JPH=1,2
      CST=DPH*ETA/(24.*PI*PI*SDK*SDK)
      C22=DPH/(3.*PI)
      SGN=-1.
      DO 80 IPH=1,NPP
      CPH=COS(PH)
      SPH=SIN(PH)
      IF(IPH.GT.1)GO TO 30
      IF(JPH.GT.1)GO TO 30
      PHO=DPH/10.
      CPH=COS(PHO)
      SPH=SIN(PHO)
 30   RH=AK*SPH
      RS=2.*AK2*(1.-CPH)
      RK=DSQRT(RS)
      VF=3.+SGN
      IF(IPH.EQ.1)VF=1.
      IF(IPH.EQ.NPP)VF=1.
      VST=VF*CST
      V22=VF*C22
      DO 40 N=1,MAX2
      I=N-1
      DZ=I*DK
      DZS=DZ*DZ
      R=DSQRT(RS+DZS)
      ARG=R+DZ
      IF(N.EQ.1)ARG=RK
      CALL CISI(CP(N),CIN,SP(N),ARG)
      EP(N)=CMPLX(CP(N),-SP(N))
      IF(N.GT.1)GO TO 38
      CM(1)=CP(1)
      SM(1)=SP(1)
      EM(1)=EP(1)
      GO TO 40
 38   ARG=RS/ARG
      CALL CISI(CM(N),CIN,SM(N),ARG)
      EM(N)=CMPLX(CM(N),-SM(N))
 40   CONTINUE
      R=4.*(-CM(2)+2.*CP(1)-CP(2))
     A+2.*CID(3)*(+CM(3)-2.*CM(2)+2.*CP(1)-2.*CP(2)+CP(3))
     B+2.*SID(3)*(-SM(3)+2.*SM(2)-2.*SP(2)+SP(3))
      X=4.*(SM(2)-2.*SP(1)+SP(2))
     C+2.*CID(3)*(-SM(3)+2.*SM(2)-2.*SP(1)+2.*SP(2)-SP(3))
```

```
     D+2.*SID(3)*(-CM(3)+2.*CM(2)-2.*CP(2)+CP(3))
      Z(1)=Z(1)+WST*CMPLX(R,X)
      IF(NEQ.EQ.1)GO TO 70
      R=2.*CID(2)*(-CM(3)+3.*CM(2)-4.*CP(1)+3.*CP(2)-CP(3))
     E+2.*SID(2)*(+SM(3)-2.*SM(2)+2.*SP(2)-SP(3))
     F+CID(4)*(+CM(4)-2.*CM(3)+CM(2)+CP(2)-2.*CP(3)+CP(4))
     G+SID(4)*(-SM(4)+2.*SM(3)-SM(2)+SP(2)-2.*SP(3)+SP(4))
      X=2.*CID(2)*(SM(3)-3.*SM(2)+4.*SP(1)-3.*SP(2)+SP(3))
     H +2.*SID(2)*(CM(3)-2.*CM(2)+2.*CP(2)-CP(3))
     I+CID(4)*(-SM(4)+2.*SM(3)-SM(2)-SP(2)+2.*SP(3)-SP(4))
     J+SID(4)*(-CM(4)+2.*CM(3)-CM(2)+CP(2)-2.*CP(3)+CP(4))
      Z(2)=Z(2)+WST*CMPLX(R,X)
      IF(NEQ.EQ.2)GO TO 70
      S1=DK
      DO 60 N=3,NEQ
      M1=N-1
      M2=N-2
      N1=N+1
      N2=N+2
      CPA=CP(M2)-2.*CP(M1)+CP(N)
      CPB=2.*CP(N)-CP(M1)-CP(N1)
      CPC=CP(N2)-2.*CP(N1)+CP(N)
      CMA=CM(M2)-2.*CM(M1)+CM(N)
      CMB=2.*CM(N)-CM(N1)-CM(M1)
      CMC=CM(N2)-2.*CM(N1)+CM(N)
      SPA=SP(M2)-2.*SP(M1)+SP(N)
      SPB=2.*SP(N)-SP(M1)-SP(N1)
      SPC=SP(N2)-2.*SP(N1)+SP(N)
      SMA=SM(M2)-2.*SM(M1)+SM(N)
      SMB=2.*SM(N)-SM(N1)-SM(M1)
      SMC=SM(N2)-2.*SM(N1)+SM(N)
      R=CID(M2)*(CPA+CMA)+2.*CID(N)*(CPB+CMB)+2.*SID(N)*(SPB-SMB)
     K +CID(N2)*(CPC+CMC)+SID(N2)*(SPC-SMC)
      IF(N.GT.3)R=R+SID(M2)*(SPA-SMA)
      X=-CID(M2)*(SPA+SMA)-2.*CID(N)*(SPB+SMB)+2.*SID(N)*(CPB-CMB)
     L -CID(N2)*(SPC+SMC)+SID(N2)*(CPC-CMC)
      IF(N.GT.3)X=X+SID(M2)*(CPA-CMA)
   60 Z(N)=Z(N)+WST*CMPLX(R,X)
   70 PH=PH+DPH
   80 SGN=-SGN
      DPH=(PI-PHA)/NPH
  100 PH=PHA
      RETURN
      END
C
C
C*****************************************************************************
C          SUBROUTINE ZSURF                                                  *
C*****************************************************************************
      SUBROUTINE ZSURF(AK,CMM,FMC,ZS)
C*****************************************************************************
C     ZSURF calculates the surface impedance ZS for thin wire.               *
C     AK = k*a where k = 2*pi/lambda and a = wire radius.                    *
C     CMM = conductivity of wire (megamhos/meter)                            *
C     CMM = -1 for perfect conductivity.                                     *
C     FMC = frequency (megahertz).                                           *
C*****************************************************************************
      COMPLEX BES,BES1,ZS
      DATA ETA,SQT,TP/376.72727,1.41421356,6.2831853/
      SQSWB=1.E6*SQRT(CMM/TP/FMC/8.85433)
```

```
      X=AK*SQSWE
      IF(X.GT.8.)GO TO 50
      T=X/8.
      T2=T*T
      T4=T2*T2
      BER=((((((-.901E-5*T4+.122552E-2)*T4-.08349609)*T4
     2+2.641914)*T4-32.363456)*T4+113.77778)*T4-64.)*T4+1.
      BEI=((((((.11346E-3*T4-.01103667)*T4+.52185615)*T4
     2-10.567658)*T4+72.817777)*T4-113.77778)*T4+16.)*T2
      BERP=X*T2*((((((-.394E-5*T4+.45957E-3)*T4-.02609253)*T4
     2+.66047849)*T4-6.0681481)*T4+14.222222)*T4-4.)
      BEIP=X*((((((.4609E-4*T4-.379386E-2)*T4+.14677204)*T4
     2-2.3116751)*T4+11.377778)*T4-10.666667)*T4+.5)
      BES=CMPLX(BER,BEI)
      BES1=.707107*CMPLX(BERP-BEIP,BERP+BEIP)
      GO TO 100
   50 XP=.70710681*X
      X1=1./X
      F=((-.0459205*X1+.390625E-2)*X1+.08838835)*X1+1.
      T=((-.04603559*X1-.0625)*X1-.08838835)*X1-.39269907+XP
      BES=F*CMPLX(COS(T),SIN(T))
      F=((.11290231*X1+.03515625)*X1-.26516505)*X1+1.
      T=((.1160097*X1+.1875)*X1+.26516505)*X1+1.1780972+XP
      BES1=F*CMPLX(COS(T),SIN(T))
  100 ZS=-CMPLX(1.,-1.)*ETA*BES/BES1/SQT/SQSWE
      RETURN
      END
C
```

APPENDIX D

COMPUTER PROGRAM WAIT-SURTEES FOR THE INPUT IMPEDANCE OF A MONOPOLE ELEMENT ON A DISK GROUND PLANE ABOVE FLAT EARTH

COMPUTER PROGRAM WAIT-SURTEES
by
JACK H. RICHMOND
December 29, 1989

INTRODUCTION [1]

Appendix I presents Richmond's computer program WAIT-SURTEES.FOR together with the subroutines CISI, FRILLS, TPLZ, TSPAR and ZSURF. This FORTRAN program calculates the impedance of a *vertical monopole* antenna centered on a circular disk* over the flat lossy earth. This program combines the following:

a) Richmond's moment method for the impedance Z_∞ of a vertical monopole antenna on an infinite ground plane,* and

b) The theory of Wait and Surtees for the change $\triangle Z$ in the antenna impedance, where $\triangle Z = Z_f - Z_\infty$ and Z_f denotes the impedance of the vertical monopole on a finite circular ground plane over the flat earth.

See: [J. R. Wait and W. J. Surtees, "Impedance of Top-Loaded Antenna of Arbitrary Length Over a Circular Grounded Screen," J. Appl. Phys., Vol. 25, pp. 553-555, May 1954].

Comment statements have been inserted in the main computer program and in each subroutine to assist the user. Only a few brief additional comments will be required in this Introduction.

In calculating Z_∞, the monopole is divided into segments of equal length and the unknown current distribution is expanded in overlapping sinusoidal basis functions. Thus, $I(z)$ is taken to be piecewise sinusoidal. The magnetic-frill model is employed (rather than the slice-generator model), and boundary matching is enforced on the surface of the monopole rather than on the axis. The wire radius is assumed to be much smaller than the wavelength. The wire monopole may be assigned perfect conductivity or finite conductivity as desired. With Galerkin's method, the calculated impedance Z_∞ is believed to be accurate for short, medium and long

[1] **Appreciation is expressed to The MITRE Corporation for sponsoring this report. The computer program WAIT-SURTEES.FOR was developed by Richmond in 1979 with other sponsorship.**

* of infinite conductivity

monopoles. If the monopole length exceeds 6 wavelengths, however, one may wish to increase the dimensions (IDM=99) in the main program and subroutine TSPAR.

In the theory of Wait and Surtees, the monopole is assumed to have a sinusoidal current distribution. (*It does not appear difficult to generalize this to a piecewise-sinusoidal distribution, but we have not attempted this.*) Since the current distribution departs significantly from the sinusoidal form when the monopole length exceeds one-half wavelength, $\triangle Z$ and Z_f may begin to lose reliability as the monopole length increases. We have not investigated this possible problem.

Appendix II presents the output data generated by WAIT-SURTEES.FOR on a VAX computer with the same input data indicated in Appendix I. This output shows excellent agreement with the original results obtained in 1979 on a DATACRAFT computer. This indicates that no additional double-precision operations are required for VAX operation.

Richmond has shown that WAIT-SURTEES.FOR is useful even for a monopole antenna on a circular disk in free space. [J. H. Richmond, "Monopole Antenna on Circular Disk," IEEE Trans., Vol. AP-32, pp. 1282-1287, December 1984]. For this case, set ER=1 and SIG=0 in the main program.

In WAIT-SURTEES.FOR the monopole is centered on a circular disk which may lie on the surface of the earth, or it may be located any distance above the earth surface. In the main program HDL denotes the height of the circular disk above the flat earth, measured in free-space wavelengths.

For a monopole on a circular disk on the surface of the earth, Richmond has shown satisfactory agreement between WAIT-SURTEES.FOR and Richmond's moment method (which enforces the boundary conditions to determine the current distributions on the monopole and the disk). See: [J. H. Richmond, "Monopole Antenna on Circular Disk Over Flat Earth," IEEE Trans., Vol. AP-33, pp. 633-637, June 1985.]

In the output data of Appendix II, the resistance RFIN is plotted as the dashed-line curve of Figure 5 in [Richmond, 1985]. The reactance XFIN in Appendix II should have been plotted as the dashed-line curve of Figure 6 in [Richmond, 1985]. By mistake, however, the dashed-line curve of Figure 6 shows the output of WAIT-SURTEES.FOR for a monopole on a circular disk in free space.

Appendix I. WAIT-SURTEES.FOR

```
C                             WAIT-SURTEES.FOR                        WAIT.1
C      IMPEDANCE OF MONOPOLE AT CENTER OF CIRCULAR DISK ON FLAT EARTH.
C      SEE: WAIT AND SURTEES, "IMPEDANCE OF TOP-LOADED ANTENNA OF
C        ARBITRARY LENGTH OVER A CIRCULAR GROUNDED SCREEN," J. APPL. PHYS.,
C        VOL. 25, PP. 553-555, MAY 1954.
C      LINK CISI;FRILLS;TPLZ;TSPAR;ZSURF
       COMPLEX CQQ,CJI,CJA,CJB,DELZ
       COMPLEX EC,EGZ,EJH,EJS,ETA2,ETD,ETH
       COMPLEX GM,GP,Q1,Q2,Q3,RC
       COMPLEX Y11,Z11,ZFIN,ZINF,ZS,ZSG
       COMPLEX CJ(99),GI(99),U(99),VJ(99),W(99),ZJ(99)
       DATA ETA,PI,TP/376.730366239,3.14159265359,6.28318530718/
       DATA E0,U0/8.85418533677E-12,1.25663706144E-6/
       DATA P2/1.57079632679/
       DATA IDM/99/
C      AI,AL,AM = RADIUS OF MONOPOLE WIRE IN (INCHES,WAVELENGTHS,METERS).
C      BAR      = RATIO OF OUTER RADIUS AND INNER RADIUS OF COAXIAL FEED.
C      BL,BM    = OUTER RADIUS OF CIRCULAR DISK (WAVELENGTHS,METERS).
C      CMM      = CONDUCTIVITY (MEGAMHOS/METER) OF MONOPOLE WIRE.
C      CMM      = -1. FOR PERFECTLY CONDUCTING MONOPOLE.
C      ER       = RELATIVE PERMITTIVITY OF EARTH.
C      FMC      = FREQUENCY (MEGAHERTZ).
C      HDL      = HEIGHT OF CIRCULAR DISK ABOVE THE FLAT EARTH (WAVELENGTHS).
C      HDL      = -1. FOR CIRCULAR DISK ON THE SURFACE OF THE EARTH.
C      HL,HM    = LENGTH OF MONOPOLE (WAVELENGTHS,METERS)
C      SIG      = CONDUCTIVITY OF EARTH (MHOS/METER).
C      WAVM     = WAVELENGTH IN FREE-SPACE (METERS).
  2    FORMAT(1X,8F11.5)
  5    FORMAT(1H0)
       AL=.003
       BAR = 3.
       CMM=-1.
       ER=4.
       FMC=300.
       HL=.229
       SIG=.001
       WAVM=300./FMC
       IWR=1
       OMEG=TP*FMC*1.E6
       EC=CMPLX(ER,-SIG/(OMEG*E0))
       ETA2=ETA/CSQRT(EC)
       AK=TP*AL
       HK=TP*HL
       SK=2.*HK
       EJS=CMPLX(COS(SK),SIN(SK))
       CHK=COS(HK)
       SHK=SIN(HK)
       EJH=CMPLX(CHK,SHK)
       NS=15.*HL
       IF(NS.LT.6)NS=6
       IF(NS.GT.IDM)NS=IDM
       IG=NS/2
       NS=2*IG
       N=NS-1
       ZS=(.0,.0)
       IF(CMM.GT.0.)CALL ZSURF(AK,CMM,FMC,ZS)
C      ZS = SURFACE IMPEDANCE OF MONOPOLE WIRE.
       DK=HK/IG
       CDK=COS(DK)
       SDK=SIN(DK)
C      ZJ(N) = FIRST ROW OF IMPEDANCE MATRIX FOR DIPOLE IN FREE SPACE,
C      USING GALERKIN'S METHOD WITH OVERLAPPING SINUSOIDAL BASIS FUNCTIONS.
C      (DIPOLE LENGTH = TWICE THE MONOPOLE LENGTH, USING IMAGE THEORY.)
       CALL TSPAR(AK,DK,N,ZJ)
       IF(CMM.LT.0.)GO TO 52
       GK=2.*(DK-CDK*SDK)
```

```
      GL=SDK-DK*CDK
      FH=4.*PI*AK*SDK*SDK
      ZJ(1)=ZJ(1)+ZS*GK/FH
      ZJ(2)=ZJ(2)+ZS*GL/FH
   52 I12=1
C     SET UP THE MOMENT-METHOD VOLTAGE COLUMN VJ(M) FOR DIPOLE IN FREE SPACE.
      CALL FRILLS(AK,BAR,DK,N,CJ)
      DO 60 I=1,N
      K=1+IABS(IG-I)
   60 VJ(I)=CJ(K)
C     SOLVE THE SIMULTANEOUS LINEAR EQUATIONS TO DETERMINE THE CURRENTS CJ(N)
C     ON THE WIRE DIPOLE IN FREE SPACE.  (THE IMPEDANCE MATRIX IS TOEPLITZ.)
      CALL TPLZ(CJ,U,VJ,W,ZJ,IER,IWR,I12,N)
C     ZINF = IMPEDANCE OF MONOPOLE ANTENNA OVER INFINITE GROUND PLANE.
      ZINF=.5/CJ(IG)
      WRITE(6,2)AL,HL,ZINF
        WRITE(15,2)AL,HL,ZINF
      WRITE(6,5)
        WRITE(15,5)
      HDL=-1.
      HDK=TP*HDL
      ZSG=ETA2
      IF(HDL.LE..0)GO TO 70
      RC=(ETA2-ETA)/(ETA2+ETA)
      EJH=CMPLX(COS(HDK),SIN(HDK))
      ZSG=ETA*(EJH+RC/EJH)/(EJH-RC/EJH)
   70 CONTINUE
      DO 100 IBL=1,16
      BL=.1*IBL
      BK=TP*BL
      RK=SQRT(BK*BK+HK*HK)
C     ZINF = IMPEDANCE OF MONOPOLE ON INFINITE CIRCULAR DISK.
C     ZFIN = IMPEDANCE OF MONOPOLE AT CENTER OF FINITE CIRCULAR DISK.
C     DELZ = ZFIN - ZINF.
C     CALCULATE DELZ USING THE FORMULA OF WAIT AND SURTEES.
      CALL CISI(CP,CIN,SP,2.*(RK+HK))
      CALL CISI(CM,CIN,SM,2.*(RK-HK))
      Q1=CMPLX(CP,P2-SP)*EJS+CMPLX(CM,P2-SM)/EJS
      CALL CISI(CP,CIN,SP,RK+BK+HK)
      CALL CISI(CM,CIN,SM,RK+BK-HK)
      CALL CISI(CB,CIN,SB,RK+BK)
      Q2=CMPLX(CP,P2-SP)*EJH+CMPLX(CM,P2-SM)/EJH-CMPLX(CB,P2-SB)
      CALL CISI(CI,CIN,SI,2.*BK)
      Q3=CMPLX(CI,P2-SI)
      DELZ=Q1-4.*CHK*Q2+2.*CHK*CHK*Q3
      DELZ=ZSG*DELZ/(4.*PI*SHK*SHK)
      ZFIN=ZINF+DELZ
        WRITE(15,2)BL,ZFIN
  100 WRITE(6,2)BL,ZFIN
  400 CALL EXIT
      END
C
```

```
C                                                                        CISI.1
      SUBROUTINE CISI(CI,CIN,SI,X)
C     Standard IBM Fortran Subroutine with slight modifications.
C     COSINE INTEGRAL AND SINE INTEGRAL.
C     X = ARGUMENT (REAL AND POSITIVE).
C     CI = Ci(x).
C     SI = Si(x).
C     CIN = Cin(x).
      DATA GAM,P2/.57721566,1.57079632/
      A=ABS(X)
      IF(A.GT.4.)GO TO 10
      IF(A.GT..1)GO TO 3
      IF(A.GT.0.)GO TO 2
      CI=.0
      CIN=.0
      SI=.0
      RETURN
  2   X2=A*A
      SI=X*((.03*X2-1.)*X2/18.+1.)
      CIN=.25*X2*((X2/45.-1.)*X2/24.+1.)
      GO TO 8
  3   Y=(4.-A)*(4.+A)
      SI=X*(((((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)*Y+6.939889E-4)
     C*Y+1.964882E-2)*Y+4.395509E-1)
      CIN=                         A*A*(((((1.386985E-10*Y+1.584996E-8)*Y
     C+1.725752E-6)*Y+1.185999E-4)*Y+4.990920E-3)*Y+1.315308E-1)
  8   CI=GAM+ALOG(A)-CIN
      RETURN
  10  SI=SIN(A)
      Y=COS(A)
      Z=4./A
      U=((((((((4.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)*Z-7.261642E-2)
     C*Z+4.987716E-2)*Z-3.332519E-3)*Z-2.314617E-2)*Z-1.134958E-5)*Z
     C+6.250011E-2)*Z+2.583989E-10
      V=(((((((((-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z
     C+7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+2.601293E-2)*Z
     C-3.764000E-4)*Z-3.122418E-2)*Z-6.646441E-7)*Z+2.500000E-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)+P2
      IF(X.LT..0)SI=-SI
      CIN=GAM+ALOG(A)-CI
      RETURN
      END
C
```

```
C                                                                    FRILLS.1
      SUBROUTINE FRILLS(AK,BAR,DK,NEQ,VJ)
C     FRILLS sets up the voltage column VJ(I).
C     VJ(I) = voltage column for perfectly conducting wire dipole in
C       free space using Galerkin's method and sinusoidal bases, and
C       matching the boundary conditions on the surface of the wire.
C     Using magnetic-frill model for center-fed dipole.
      REAL*8 DZS,RS1,RS2
      COMPLEX EGZ,GM,GP,GI(20),VJ(1),GII,QST,WST
      DATA PI,TP/3.14159265359,6.28318530718/
      IDM=20
      DO 20 I=1,NEQ
   20 VJ(I)=(.0,.0)
      VJ(1)=(1.,.0)
      IF(BAR.LE.1.)RETURN
      VJ(1)=(.0,.0)
      NSW=NEQ+1
      SDK=SIN(DK)
      CDK=COS(DK)
      BAL=ALOG(BAR)
      QST=CMPLX(.0,1./(4.*BAL*SDK))
      BK=AK*BAR
      AKS=AK*AK
      BKS=BK*BK
      LIM=NSW+1
      IF(LIM.GT.IDM)LIM=IDM
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      PH=.0
      DO 90 LPH=1,2
      WST=DPH*QST/(3.*PI)
      SGN=-1.
      DO 80 IPH=1,NPP
      WF=3.+SGN
      IF(IPH.EQ.1)WF=1.
      IF(IPH.EQ.NPP)WF=1.
      CPH=COS(PH)
      IF(IPH.GT.1)GO TO 40
      IF(LPH.GT.1)GO TO 40
      CPH=COS(DPH/10.)
   40 RS1=2.*AKS*(1.-CPH)
      RS2=AKS+BKS-2.*AK*BK*CPH
      RH1=DSQRT(RS1)
      RH2=DSQRT(RS2)
      CALL CISI(CA,CIN,SA,RH1)
      CALL CISI(CB,CIN,SB,RH2)
      GI(1)=2.*CMPLX(CB-CA,SA-SB)
      DO 50 I=2,LIM
      DZ=DK*(I-1)
      DZS=DZ*DZ
      RA=DSQRT(RS1+DZS)
      RB=DSQRT(RS2+DZS)
      CALL CISI(C1,CIN,S1,RA+DZ)
      CALL CISI(C2,CIN,S2,RB+DZ)
      GP=CMPLX(C2-C1,S1-S2)
      RAM=RS1/(RA+DZ)
      RBM=RS2/(RB+DZ)
      CALL CISI(C1,CIN,S1,RAM)
      CALL CISI(C2,CIN,S2,RBM)
      GM=CMPLX(C2-C1,S1-S2)
      EGZ=CMPLX(COS(DZ),SIN(DZ))
   50 GI(I)=GP*EGZ+GM/EGZ
      VJ(1)=VJ(1)+WF*WST*(GI(2)-CDK*GI(1))
```

```
      IF(NEQ.LE.1)GO TO 78
      K1=0
      DO 60 I=2,NEQ
      K1=K1+1
      K2=K1+1
      K3=K2+1
      IF(K3.GT.IDM)GO TO 60
      GP=GI(K1)-2.*CDK*GI(K2)+GI(K3)
      VJ(I)=VJ(I)+WF*WST*GP
   60 CONTINUE
   78 SGN=-SGN
   80 PH=PH+DPH
      DPH=(PI-PHA)/NPH
   90 PH=PHA
      VJ(1)=2.*VJ(1)
      RETURN
      END
C
```

```
C
      SUBROUTINE TPL2(C,U,VJ,W,Z,IER,IWR,I12,NEQ)
C       MODIFIED VERSION OF SUBROUTINE FURNISHED BY CHARLES KLEIN.
C       SOLVES SIMULTANEOUS LINEAR EQUATIONS.
C       SET IWR =(1 OR 0) TO GET (PRINTOUT OR NO-PRINTOUT).
C       SET I12 = 1 ON FIRST CALL, WHERE MATRIX INVERSION IS REQUIRED.
C       SET I12 = 2 IF MATRIX Z HAS ALREADY BEEN INVERTED ON PREVIOUS CALL.
C       NEQ = NUMBER OF SIMULTANEOUS LINEAR EQUATIONS.
C  Z(J) IS THE FIRST ROW OF THE TOEPLITZ IMPEDANCE MATRIX
C  VJ(J) = INPUT VOLTAGE COLUMN
C  C(J) = OUTPUT CURRENT COLUMN
C  U(J) AND W(J) ARE WORK ARRAYS OF LENGTH NEQ
C  IF IER = 0 , NO ERROR OCCURRED
      COMPLEX C(1),U(1),VJ(1),W(1),Z(1)
      COMPLEX ALMDA,ALPHA,C1,C2,COEF,FAC,TAU1,V,V1,V2
  2   FORMAT(1X,I5,F10.3,F15.7,F10.1)
  5   FORMAT(1H0)
      IF(NEQ.GT.1)GO TO 8
      C(1)=VJ(1)/Z(1)
      CNOR=CABS(C(1))
      GO TO 100
  8   IF(I12.NE.1)GO TO 45
      N=NEQ-1
      IER=0
C  NORMALIZE INPUT MATRIX
      TAU1=Z(1)
      DO 10 II=1,N
 10   Z(II)=Z(II+1)/TAU1
      ALMDA=1.-Z(1)*Z(1)
      U(1)=-Z(1)
      I=2
 15   KK=I-1
      ALPHA=(.0,.0)
      DO 20 M=1,KK
      LL=I-M
 20   ALPHA=ALPHA+U(M)*Z(LL)
      ALPHA=-(ALPHA+Z(I))
      IF(CABS(ALPHA).EQ..0)GO TO 130
      COEF=ALPHA/ALMDA
      ALMDA=ALMDA-COEF*ALPHA
      DO 30 J=1,KK
      L=I-J
 30   W(J)=U(J)+COEF*U(L)
      DO 40 J=1,KK
 40   U(J)=W(J)
      U(I)=COEF
      IF(I.GE.N)GO TO 45
      I=I+1
      GO TO 15
C  THE FOLLOWING COMPUTES THE ELEMENTS OF THE INVERSE
 45   NH=(NEQ+1)/2
      FAC=ALMDA*TAU1
      NP=NEQ+1
      CNOR=.0
      DO 90 I=1,NH
      IF(I.NE.1)GO TO 55
      W(1)=1./FAC
      DO 50 J=2,NEQ
 50   W(J)=U(J-1)/FAC
      GO TO 70
 55   C1=U(I-1)
      NPI=NP-I
      C2=U(NPI)
      DO 60 JJ=1,N
      J=NP-JJ
      NPJ=NP-J
```

8

```
   60  W(J)=W(J-1)+(C1*U(J-1)-C2*U(NPJ))/FAC
       W(1)=U(I-1)/FAC
C  MATRIX MULTIPLY
   70  V=(.0,.0)
       V1=(.0,.0)
       DO 80 J=1,NEQ
       V2=VJ(J)
       V=V+V2*W(J)
       NPJ=NP-J
   80  V1=V1+V2*W(NPJ)
       C(I)=V
       NPI=NP-I
       C(NPI)=V1
       IF(IWR.LE.0)GO TO 90
       CA=CABS(V)
       IF(CA.GT.CNOR)CNOR=CA
       CA=CABS(V1)
       IF(CA.GT.CNOR)CNOR=CA
   90  CONTINUE
  100  IF(IWR.LE.0)GO TO 120
C  PRINT OUT THE SOLUTION FOR THE CURRENTS  C(J)
       WRITE(6,5)
       IF(CNOR.LE.0.)CNOR=1.
       DO 110 I=1,NEQ
       V=C(I)
       CA=CABS(V)
       CN=CA/CNOR
       PH=.0
       IF(CA.GT.0.)PH=57.29578*ATAN2(AIMAG(V),REAL(V))
  110  WRITE(6,2)I,CN,CA,PH
       WRITE(6,5)
  120  RETURN
  130  IER=I
       RETURN
       END
C
```

9

```
C
      SUBROUTINE TSPAR(AK,DK,NEQ,Z)
C     TSPAR sets up the impedance matrix Z(J).
C     Z(J) - First row of impedance matrix for perfectly conducting
C       thin-wire dipole in free space, using Galerkin's method with
C       overlapping sinusoidal basis functions and matching the
C       boundary conditions on the surface of the wire.
C     AK - k*a, where k - 2*pi/lambda and a - wire radius.
C     DK - k*d, where d - segment length.
C     NEQ - number of simultaneous linear equations.
      REAL*8 DZS,RS
      COMPLEX EID(90),EM(90),EP(90),Z(1)
      COMPLEX CEM,CEP,EMD,EPD,EMD2,EPD2,Z11,Z22,G11,Q11
      DIMENSION CID(90),SID(90),CM(90),CP(90),SM(90),SP(90)
      DATA GAM,P2/.577215664,1.57079632/
      DATA ETA,PI/376.727,3.14159/
      IDM=90
    1 FORMAT(3X,'MUST INCREASE DIMENSIONS IN SUBROUTINE TSPAR')
    2 FORMAT(3X,'ACTUAL DIMENSION IDM - ',I5,6X,
     2'REQUIRED DIMENSION MAX2 - ',I5)
      IF(NEQ.LE.0)RETURN
      MAX2=NEQ+2
      DO 14 I=1,NEQ
   14 Z(I)=(.0,.0)
      IF(MAX2.LE.IDM)GO TO 16
      WRITE(6,1)
      WRITE(6,2)IDM,MAX2
      RETURN
   16 TDK=2.*DK
      S11=.0
      S13=TDK
      S21=DK
      S23=3.*DK
      DO 20 N=1,MAX2
      I=N-1
      DZ=I*DK
      CID(N)=COS(DZ)
      SID(N)=SIN(DZ)
   20 EID(N)=CMPLX(CID(N),SID(N))
      CDK=COS(DK)
      SDK=SIN(DK)
      EPD=CMPLX(CDK,SDK)
      EMD=CMPLX(CDK,-SDK)
      EPD2=EPD*EPD
      EMD2=EMD*EMD
      CEM=2.*CDK+EMD
      CEP=2.*CDK+EPD
      AK2=AK*AK
      CSS=ETA/(8.*PI*SDK*SDK)
      NPH=6
      NPH=2*(NPH/2)
      NPP=NPH+1
      PHA=.0174533*20.
      DPH=PHA/NPH
      PH=.0
      DO 100 JPH=1,2
      CST=DPH*ETA/(24.*PI*PI*SDK*SDK)
      C22=DPH/(3.*PI)
      SGN=-1.
      DO 80 IPH=1,NPP
      CPH=COS(PH)
      SPH=SIN(PH)
      IF(IPH.GT.1)GO TO 30
      IF(JPH.GT.1)GO TO 30
      PH0=DPH/10.
      CPH=COS(PH0)
```

```
      SPH=SIN(PHO)
   30 RH=AK*SPH
      RS=2.*AK2*(1.-CPH)
      RK=DSQRT(RS)
      WF=3.+SGN
      IF(IPH.EQ.1)WF=1.
      IF(IPH.EQ.NPP)WF=1.
      WST=WF*CST
      W22=WF*C22
      DO 40 N=1,MAX2
      I=N-1
      DZ=I*DK
      DZS=DZ*DZ
      R=DSQRT(RS+DZS)
      ARG=R+DZ
      IF(N.EQ.1)ARG=RK
      CALL CISI(CP(N),CIN,SP(N),ARG)
      EP(N)=CMPLX(CP(N),-SP(N))
      IF(N.GT.1)GO TO 38
      CM(1)=CP(1)
      SM(1)=SP(1)
      EM(1)=EP(1)
      GO TO 40
   38 ARG=RS/ARG
      CALL CISI(CM(N),CIN,SM(N),ARG)
      EM(N)=CMPLX(CM(N),-SM(N))
   40 CONTINUE
      R=4.*(-CM(2)+2.*CP(1)-CP(2))
     A+2.*CID(3)*(+CM(3)-2.*CM(2)+2.*CP(1)-2.*CP(2)+CP(3))
     B+2.*SID(3)*(-SM(3)+2.*SM(2)-2.*SP(2)+SP(3))
      X=4.*(SM(2)-2.*SP(1)+SP(2))
     C+2.*CID(3)*(-SM(3)+2.*SM(2)-2.*SP(1)+2.*SP(2)-SP(3))
     D+2.*SID(3)*(-CM(3)+2.*CM(2)-2.*CP(2)+CP(3))
      Z(1)=Z(1)+WST*CMPLX(R,X)
      IF(NEQ.EQ.1)GO TO 70
      R=2.*CID(2)*(-CM(3)+3.*CM(2)-4.*CP(1)+3.*CP(2)-CP(3))
     E+2.*SID(2)*(+SM(3)-2.*SM(2)+2.*SP(2)-SP(3))
     F+CID(4)*(+CM(4)-2.*CM(3)+CM(2)+CP(2)-2.*CP(3)+CP(4))
     G+SID(4)*(-SM(4)+2.*SM(3)-SM(2)+SP(2)-2.*SP(3)+SP(4))
      X=2.*CID(2)*(SM(3)-3.*SM(2)+4.*SP(1)-3.*SP(2)+SP(3))
     H +2.*SID(2)*(CM(3)-2.*CM(2)+2.*CP(2)-CP(3))
     I+CID(4)*(-SM(4)+2.*SM(3)-SM(2)-SP(2)+2.*SP(3)-SP(4))
     J+SID(4)*(-CM(4)+2.*CM(3)-CM(2)+CP(2)-2.*CP(3)+CP(4))
      Z(2)=Z(2)+WST*CMPLX(R,X)
      IF(NEQ.EQ.2)GO TO 70
      S1=DK
      DO 60 N=3,NEQ
      M1=N-1
      M2=N-2
      N1=N+1
      N2=N+2
      CPA=CP(M2)-2.*CP(M1)+CP(N)
      CPB=2.*CP(N)-CP(M1)-CP(N1)
      CPC=CP(N2)-2.*CP(N1)+CP(N)
      CMA=CM(M2)-2.*CM(M1)+CM(N)
      CMB=2.*CM(N)-CM(N1)-CM(M1)
      CMC=CM(N2)-2.*CM(N1)+CM(N)
      SPA=SP(M2)-2.*SP(M1)+SP(N)
      SPB=2.*SP(N)-SP(M1)-SP(N1)
      SPC=SP(N2)-2.*SP(N1)+SP(N)
      SMA=SM(M2)-2.*SM(M1)+SM(N)
      SMB=2.*SM(N)-SM(N1)-SM(M1)
      SMC=SM(N2)-2.*SM(N1)+SM(N)
      R=CID(M2)*(CPA+CMA)+2.*CID(N)*(CPB+CMB)+2.*SID(N)*(SPB-SMB)
     R +CID(N2)*(CPC+CMC)+SID(N2)*(SPC-SMC)
      IF(N.GT.3)R=R+SID(M2)*(SPA-SMA)
```

```
      X=-CID(M2)*(SPA+SMA)-2.*CID(N)*(SPB+SMB)+2.*SID(N)*(CPB-CMB)     TSPAR.3
     L -CID(N2)*(SPC+SMC)+SID(N2)*(CPC-CMC)
      IF(N.GT.3)X=X+SID(M2)*(CPA-CMA)
   60 Z(N)=Z(N)+WST*CMPLX(R,X)
   70 PH=PH+DPH
   80 SGN=-SGN
      DPH=(PI-PHA)/NPH
  100 PH=PHA
      RETURN
      END
C
```

```
C                                                                    ZSURF.1
      SUBROUTINE ZSURF(AK,CMM,FMC,ZS)
C     ZSURF calculates the surface impedance ZS for thin wire.
C     AK = k*a where k = 2*pi/lambda and a = wire radius.
C     CMM = conductivity of wire (megamhos/meter)
C     CMM = -1 for perfect conductivity.
C     FMC = frequency (megahertz).
      COMPLEX BES,BES1,ZS
      DATA ETA,SQT,TP/376.72727,1.41421356,6.2831853/
      SQSWE=1.E6*SQRT(CMM/TP/FMC/8.85433)
      X=AK*SQSWE
      IF(X.GT.8.)GO TO 50
      T=X/8.
      T2=T*T
      T4=T2*T2
      BER=((((((-.901E-5*T4+.122552E-2)*T4-.08349609)*T4
     2+2.641914)*T4-32.363456)*T4+113.77778)*T4-64.)*T4+1.
      BEI=((((((.11346E-3*T4-.01103667)*T4+.52185615)*T4
     2-10.567658)*T4+72.817777)*T4-113.77778)*T4+16.)*T2
      BERP=X*T2*((((((-.394E-5*T4+.45957E-3)*T4-.02609253)*T4
     2+.66047849)*T4-6.0681481)*T4+14.222222)*T4-4.)
      BEIP=X*((((((.4609E-4*T4-.379386E-2)*T4+.14677204)*T4
     2-2.3116751)*T4+11.377778)*T4-10.666667)*T4+.5)
      BES=CMPLX(BER,BEI)
      BES1=.707107*CMPLX(BERP-BEIP,BERP+BEIP)
      GO TO 100
 50   XP=.70710681*X
      X1=1./X
      F=((-.0459205*X1+.390625E-2)*X1+.08838835)*X1+1.
      T=((-.04603559*X1-.0625)*X1-.08838835)*X1-.39269907+XP
      BES=F*CMPLX(COS(T),SIN(T))
      F=((.11290231*X1+.03515625)*X1-.26516505)*X1+1.
      T=((.1160097*X1+.1875)*X1+.26516505)*X1+1.1780972+XP
      BES1=F*CMPLX(COS(T),SIN(T))
 100  ZS=-CMPLX(1.,-1.)*ETA*BES/BES1/SQT/SQSWE
      RETURN
      END
```

Appendix II. Output data from Wait-Surtees.For

AL	HL	RINF	XINF
0.00300	0.22900	32.22021	-9.93279

BL	RFIN	XFIN
0.10000	43.93178	-27.96955
0.20000	28.43193	-20.13158
0.30000	25.36656	-11.10953
0.40000	29.26224	-5.85667
0.50000	34.19707	-6.53444
0.60000	35.41125	-10.37363
0.70000	32.85429	-12.58671
0.80000	30.25837	-11.25765
0.90000	30.52595	-8.70713
1.00000	32.74195	-8.13379
1.10000	33.91415	-9.83655
1.20000	32.81260	-11.36605
1.30000	31.15025	-10.87897
1.40000	31.06938	-9.27844
1.50000	32.45232	-8.72108
1.60000	33.36460	-9.77519